MINI-PROJECTS ON QUANTUM COMPUTING

1. CONSTRAINED OPTIMIZATION
2. STATE PREPARATION
3. ERROR CORRECTION

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MINI-PROJECT 2: OBJECTIVES

- Given a 2^n-dimensional state vector

$$\psi = \sum_{x=0}^{2^n-1} \psi_x |x\rangle_n \in \mathbb{C}^{2^n}$$

  such that $\|\psi\|_2 = 1$, write a function QSP (ψ) which outputs a circuit U, such that

$$U|0\rangle_n = \sum_{x=0}^{2^n-1} \psi_x |x\rangle_n \ .$$

- The construction may use any number of ancillas, arbitrary 1-qubit gates and multi-controlled R_Z gates, (the number of controlled may be arbitrarily large).
- No classical bit and measurements allowed.

THE CONSTRUCTION

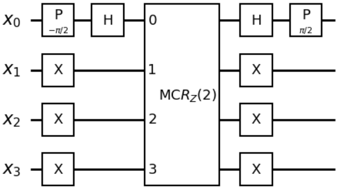
- Using $|\psi_x|$, find angles $\theta, \theta_0, \theta_1, \theta_{00}, \theta_{10}, \theta_{01}, \theta_{11}, \cdots$ such that (for example in the case of $n = 2$)

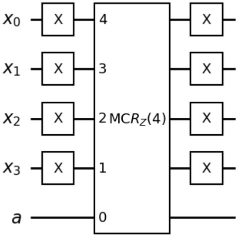
$$\sum_{x=0}^{2^2-1} |\psi_x||x\rangle_n = \cos(\theta/2)\Big(\cos(\theta_0/2)|0\rangle + \sin(\theta_0/2)|1\rangle\Big)|0\rangle$$

$$+\sin(\theta/2)\Big(\cos(\theta_1/2)|0\rangle + \sin(\theta_1/2)|1\rangle\Big)|1\rangle\ .$$

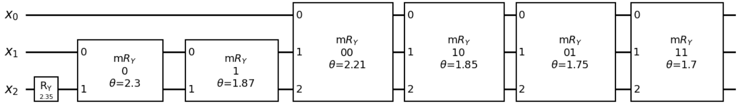
  This step is entirely classical.
- Using the R_Y multiplexers and the angles obtained in step 1, prepare the state $\sum_{x=0}^{2^n-1} |\psi_x||x\rangle_n$.
- Using the R_Z multiplexers and one ancilla, prepare the state $\sum_{x=0}^{2^n-1} \psi_x|x\rangle_n$ from the state $\sum_{x=0}^{2^n-1} |\psi_x||x\rangle_n$.

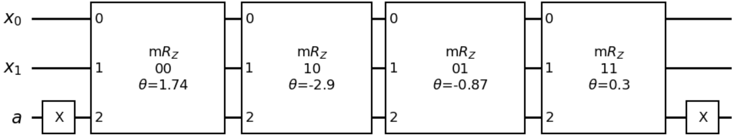
IMPLEMENTING THE R_Y AND R_Z MULTIPLEXERS



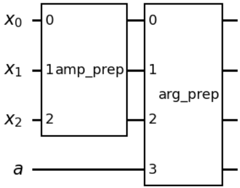


THE ABSOLUTE VALUE AND ARGUMENT STATE PREPARATION CIRCUITS





The overall circuit and its complexity



For n-qubits, the space and time complexities are as follows:

- $O(n2^n)$ 1-qubit gates,
- $O(2^n)$ multi-controlled R_Z gates,
- n qubits plus one ancilla qubit,
- $O(2^n)$ gate depth.

Thank You !