Problem 1. Evaluate the following definite integrals as limit of a sum.

1. $\displaystyle\int_0^1 x^2\,dx$.
2. $\displaystyle\int_0^2 x^3\,dx$.

Problem 2. Find the area bounded by the following curves and the x-axis, using the fundamental theorem of calculus.

1. $y = x^2 + 1,\ x = 2,\ x = 3.$
2. $y = \dfrac{2}{x^2},\ x = 1,\ x = 2.$
3. $y = x - x^2.$
4. $y = 4 - x^2.$

Problem 3. Evaluate the following integrals.

1. $\displaystyle\int x\,(2 - x^2)^4\,dx$.
2. $\displaystyle\int \frac{dt}{\sqrt{1-t}}$.
3. $\displaystyle\int \sqrt{1-2x}\,dx$.
4. $\displaystyle\int \left(\frac{2}{\sqrt{x}} - 3x\sqrt{x} + 2\right)dx$.
5. $\displaystyle\int \frac{x\,dx}{(x^2-1)^2}$.
6. $\displaystyle\int_0^4 \left(2 + \sqrt{z}\right)^2 dz$.

Answers on next page

Answers to problem 1.

1. $\Delta x_i = \frac{1}{n}$, $x_i = \frac{i}{n}$, $\int_0^1 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^2}{n^3} = \frac{n(n+1)(2n+1)}{6n^3} = \frac{1}{3}$.

2. $\Delta x_i = \frac{2}{n}$, $x_i = \frac{2i}{n}$, $\int_0^2 x^3\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{16i^3}{n^4} = 16\frac{n^2(n+1)^2}{4n^4} = 4$.

Answer to Problem 2.

1. $A = \int_2^3 (x^2+1)\,dx = \frac{22}{3}$.

2. $A = \int_1^2 \frac{2}{x^2}\,dx = 1$.

3. $x - x^2 = 0 \Rightarrow x = 0, 1 \Rightarrow A = \int_0^1 (x - x^2)\,dx = \frac{1}{6}$.

4. $4 - x^2 = 0 \Rightarrow x = \pm 2 \Rightarrow A = \int_{-2}^2 (4 - x^2)\,dx = \frac{32}{3}$.

Answers to Problem 3.

1. Substitute $u = 2 - x^2$ to get $\int x(2-x^2)^4\,dx = \frac{-1}{2}\int u^4\,du = \frac{-1}{10}(2-x^2)^5 + C$.

2. Substitute $u = 1 - t$ to get $\int \frac{dt}{\sqrt{1-t}} = -\int u^{-1/2}\,du = -2\sqrt{1-t} + C$.

3. Substitute $u = 1 - 2x$ to get $\int \sqrt{1-2x}\,dx = \frac{-1}{2}\int u^{\frac{1}{2}}\,du = \frac{-1}{3}(1-2x)^{3/2} + C$.

4. $\int \left(\frac{2}{\sqrt{x}} - 3x\sqrt{x} + 2\right) dx = 4\sqrt{x} - \frac{6}{5}x^{5/2} + 2x + C$.

5. Substitute $u = x^2 - 1$ to get $\int \frac{x\,dx}{(x^2-1)^2} = \frac{1}{2}\int u^{-2}\,du = \frac{-1}{2(x^2-1)} + C$.

6. Expand the whole square so that

$$\int_0^4 \left(2 + \sqrt{z}\right)^2 dz = \int_0^4 (4 + 4\sqrt{z} + z)\,dz = \left(4z + \frac{8}{3}z^{3/2} + \frac{1}{2}z^2\right)\Big|_0^4 = \frac{136}{3}.$$