Problem 1. Find the area bounded by the following curves.

1. $y = x,\ y = x^2$.
2. $y = 2x,\ y = 8 - x^2$.
3. $y = x^2 - 1,\ y = 3$.
4. $y = x,\ y = \sqrt{x}$.
5. $y^2 = x,\ y^3 = x$.
6. $y^2 = x - 1,\ y = x - 3$.

Problem 2. Evaluate the following improper integrals.

1. $\displaystyle\int_1^\infty \frac{3}{x^4}\,dx$.
2. $\displaystyle\int_{-1}^\infty \frac{x}{(x^2+4)^2}\,dx$.
3. $\displaystyle\int_{-\infty}^0 \frac{dt}{(2t-5)^3}$.
4. $\displaystyle\int_0^3 \frac{2}{\sqrt[3]{x-1}}\,dx$.
5. $\displaystyle\int_1^5 \frac{x}{(x^2-4)^2}\,dx$.

Problem 3. Find the area of the region enclosed by the following curves.

1. $y = 2/x^2$, $x = 3$, the x-axis to the right of $x = 3$.
2. $y = 1/(x+2)^2$ and the coordinate axes, area in the first quadrant.
3. $y = 1/\sqrt{x+4}$, $x = -4$ and the coordinate axes.

Answers on next page

Answers to problem 1.

1. $A = \int_0^1 (x - x^2)\,dx = 1/6$.
2. $A = \int_{-4}^2 (8 - x^2 - 2x)\,dx = 36$.
3. $A = \int_{-2}^2 (3 - (x^2 - 1))\,dx = 32/3$.
4. $A = \int_0^1 (\sqrt{x} - x^2)\,dx = 1/3$.
5. $A = \int_0^1 (y^2 - y^3)\,dy = 1/12$.
6. $A = \int_{-1}^2 ((y + 3) - (y^2 + 1)) = 9/2$.

Answers to problem 2.

1. Converges to 1.
2. Converges to $1/10$.
3. Converges to $-1/100$.
4. Converges to $3(2^{2/3} - 1)$.
5. Diverges.

Answers to problem 3.

1. $\displaystyle\int_3^\infty \frac{2}{x^2}\,dx = 2/3$.
2. $\displaystyle\int_0^\infty \frac{1}{(x+2)^2}\,dx = 1/2$.
3. $\displaystyle\int_{-4}^0 \frac{1}{\sqrt{x+4}}\,dx = 4$.