The derivative is the instantaneous rate of change of one variable with respect to another. When $y = f(x)$, the derivative $f'(x)$ gives the instantaneous rate of change of y with respect to x.

If moving to right of orig, +ve

Velocity:

$$v = \frac{ds}{dt} \quad \text{where } s \text{ denotes distance and } t \text{ denotes time.}$$

→ displacement → If moving to left of origin −ve

Example 1. Suppose a particle moves along the x-axis according to the equation $s = t^3 - 3t^2 - 9t + 7$. Find all the time instants at which the particle is at rest. Also, find the time intervals when the particle is moving to the right and the time intervals when the particle is moving to the left.

$s(t) = t^3 - 3t^2 - 9t + 7$

$v(t) = 3t^2 - 3(2t) - 9(1) + 0$

$= 3t^2 - 6t - 9 = 3(t^2 - 2t - 3)$

- when is $v(t) = 0$
- when is $v(t) > 0$
- when is $v(t) < 0$

$3(t^2 - 2t - 3) = 0 \Rightarrow t^2 - 2t - 3 = 0$

$\underbrace{t^2 - 3t} + \underbrace{t - 3} = 0$

$\Rightarrow t(t-3) + 1(t-3) = 0$

$\Rightarrow (t-3)(t+1) = 0$

$\Rightarrow t - 3 = 0$ or $t + 1 = 0$

$\Rightarrow t = 3$ or $\cancel{t = -1}$ time is always +ve

time instants of rest.

$\boxed{t = 3}$

$v(t) = 3(t+1)(t-3) > 0 \Rightarrow (t+1)(t-3) > 0$

$\Rightarrow t$ lies in $(-\infty, -1) \cup (3, \infty)$

$v(t) = 3(t+1)(t-3) < 0 \Rightarrow (t+1)(t-3) < 0$

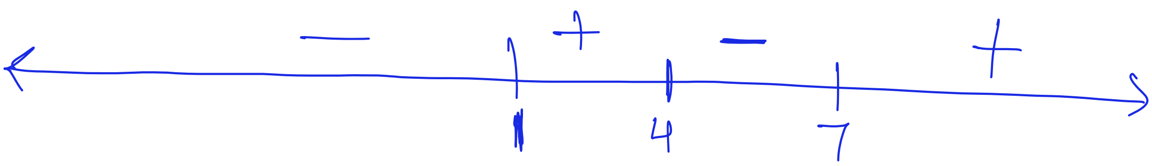
$\Rightarrow t$ lies in $(-1, 3)$

$\Rightarrow$ to the right in $(3, \infty)$

to the left in $(0, 3)$

$(t-1)(t-4)(t-7)$ → when > 0

→ when < 0



$(t-1)(t-4)(t-7) > 0 \Rightarrow (1, 4) \cup (7, \infty)$

$(t-1)(t-4)(t-7) < 0 \Rightarrow (-\infty, 1) \cup (4, 7)$

Acceleration:

$$a = \frac{dv}{dt} \quad \text{where } v \text{ denotes velocity and } t \text{ denotes time.}$$

Example 2. In example 1, find the time intervals when the particle was speeding up and the time intervals when the particle was slowing down.

$$s(t) = t^3 - 3t^2 - 9t + 7$$

$$v(t) = 3t^2 - 6t - 9$$

$$a(t) = \frac{dv}{dt} = 6t - 6 = 6(t-1)$$

<u>speeding up</u>

$a(t) > 0$

$6(t-1) > 0$

$t - 1 > 0$

$t > 1$

$(1, \infty)$

<u>slowing down</u>

$a(t) < 0$

$6(t-1) < 0$

$t - 1 < 0$

$t < 1$

$(-\infty, 1)$ reject $(-\infty, 0)$

$(0, 1)$ slowing down

1. **Current** in a circuit is given by
$$i = \frac{dq}{dt} \quad \text{where } q \text{ denotes charge and } t \text{ denotes time.}$$
2. **Voltage** across an inductor is given by
$$V = L\frac{di}{dt} \quad \text{where } i \text{ denotes current and constant } L \text{ is inductance of the inductor.}$$
3. **Charge** on a capacitor is given by $q = CV$ where C is capacitance of the capacitor and V is voltage across it, so that the current flowing through a capacitor is
$$i = C\frac{dV}{dt}.$$

Example 3. For a short time interval the current through a 0.04-H inductor is given by $i = 1.6\,t^2$. Find the voltage across the inductor at $t = 0.5$ s.

$i(t) = 1.6\,t^2$, $L = 0.04$

$V(t) = L\dfrac{di}{dt} = 0.04\,\dfrac{d}{dt}(1.6\,t^2)$

$= 0.04\,((2)(1.6)t)$

$= 0.04\,(3.2\,t)$

$V(t) = 0.128\,t$

$V(0.5) = 0.128\,(0.5) = 0.064$ V

Marginal cost: In economics, the cost incurred in producing x units of a certain commodity is called the cost function $C(x)$. The marginal cost is then given by

$$\text{marginal cost} = \frac{dC}{dx}.$$

Example 4. Suppose the cost function of a certain commodity is given by $C(x) = 2000 + 6x + 0.01x^2$. Find the average cost of producing 100 items and the marginal cost of producing one more item after having produced 100 items.

Average Cost = ?? $= \dfrac{C(100)}{100}$

Marginal Cost = ?? $\rightarrow \left.\dfrac{dC}{dx}\right|_{x=100}$

$$\frac{dC}{dx} = \frac{d}{dx}\left(2000 + 6x + 0.01x^2\right) = 6 + 0.02x$$

$$\left.\frac{dC}{dx}\right|_{x=100} = 6 + (0.02)(100) = 6 + 2 = 8\$$$

$$C(100) = 2000 + 6(100) + (0.01)(100)(100)$$
$$= 2000 + 600 + 100 = 2700$$

$$\frac{C(100)}{100} = \frac{2700}{100} = 27\$$$