Notation for higher derivatives:

$$\frac{dy}{dx}, \quad \frac{d^2y}{dx^2}, \quad \frac{d^3y}{dx^3}, \quad \frac{d^4y}{dx^4}, \cdots \frac{d^ny}{dx^n}, \cdots$$

or

$$f'(x), \quad f''(x), \quad f^{(3)}(x), \quad f^{(4)}(x), \cdots f^{(n)}(x), \cdots$$

or

$$y', \quad y'', \quad y^{(3)}, \quad y^{(4)}, \cdots y^{(n)}, \cdots$$

Example 1. Let $y = x^6 - 2x^5 - x^4$. Then find $\dfrac{d^3y}{dx^3}$.

$$\Rightarrow \frac{dy}{dx} = 6x^5 - 10x^4 - 4x^3$$

$$\Rightarrow \frac{d^2y}{dx^2} = 30x^4 - 40x^3 - 12x^2$$

$$\Rightarrow \frac{d^3y}{dx^3} = 120x^3 - 120x^2 - 24x$$

$$= 24x(5x^2 - 5x - 1)$$

Example 2. For $f(x) = \dfrac{x}{x-2}$, find $f^{(4)}(x)$.

$$f(x) = \frac{x}{x-2} = \frac{x-2+2}{x-2} = \cancel{\frac{x-2}{x-2}} + \frac{2}{x-2}$$

$$= 1 + \frac{2}{x-2} = 1 + 2(x-2)^{-1}$$

$$f'(x) = 2(-1)(x-2)^{-2}$$

$$\Rightarrow f''(x) = 2(-1)(-2)(x-2)^{-3}$$

$$\Rightarrow f^{(3)}(x) = 2(-1)(-2)(-3)(x-2)^{-4}$$

$$\Rightarrow f^{(4)}(x) = 2(-1)(-2)(-3)(-4)(x-2)^{-5}$$

$$= 48(x-2)^{-5} = \frac{48}{(x-2)^{5}}.$$