Example 1. The formula for the output P for a battery is given by

$$P = VI - RI^2 ,$$

where V is the voltage, I is the current, and R is the resistance. Find the current for which the output is a maximum if $V = 12$ volts and $R = 5$ ohms.

$P = 12I - 5I^2$

Find the max. pt of this function.

$P' = 12 - 10I \Rightarrow P'' = -10 < 0$

Critical Pts: $P' = 0 \Rightarrow 12 - 10I = 0$

$\Rightarrow I = \frac{12}{10} = 1.2$ Amperes.

Since, $P'' < 0$, $I = 1.2$ is a max. pt.

(By 2nd derivative test)

Example 2. A rectangle has an area of 100 square meter. What should be the dimensions so that the perimeter will be as small as possible.

Let the length be x and width be y.

$\Rightarrow xy = 100 \Rightarrow y = \frac{100}{x}$

$P = 2x + 2y = 2x + 2\left(\frac{100}{x}\right)$

$\Rightarrow P(x) = 2x + \frac{200}{x}$

Need to find min. pt. for $P(x)$.

$\Rightarrow P'(x) = 2 - \frac{200}{x^2} \Rightarrow P''(x) = \frac{400}{x^3}$

Critical Pts: $2 - \frac{200}{x^2} = 0 \Rightarrow \frac{200}{x^2} = 2 \Rightarrow x^2 = 100$

$\Rightarrow x = \pm 10$

$P''(10) = \frac{400}{1000} > 0$, $P''(-10) = -\frac{400}{1000} < 0 \Rightarrow x = 10$ is the reqd min. pt.

$\Rightarrow y = \frac{100}{x} = 10$

Solving minimum-maximum problems.

1. Write the expression for the quantity F to be minimized or maximized, using appropriate variables. (Drawing a figure may help.)
2. If the expression contains two variables, eliminate one of them using the information in the problem.
3. Minimize or maximize F.

Example 3. Find the number which exceeds its square by the greatest amount.

Let the number be x

and $f(x) = x - x^2$.

We want to maximize f.

$\Rightarrow f'(x) = 1 - 2x \Rightarrow f''(x) = -2 < 0$

Critical Pts. : $f'(x) = 0 \Rightarrow 1 - 2x = 0 \Rightarrow x = \frac{1}{2}$

Since, $f''(x) < 0$, by 2nd derivative test,

$x = \frac{1}{2}$ is a max. pt.

$\Rightarrow x = \frac{1}{2}$ is the required number.