Definition of Differential

$$dy = f'(x)\,dx \quad (\text{ where } dx = \Delta x = x_2 - x_1)\,.$$

Example 1. If $y = x^2$, compute Δy and dy if $x = 2$ and $dx = \Delta x = 0.01$.

$x_1 = 2$, $x_2 = 2 + \Delta x = 2.01$

$y_1 = x_1^2 = (2)^2 = 4$

$y_2 = x_2^2 = (2.01)^2 = 4.0401$

$\Delta y = y_2 - y_1 = 4.0401 - 4 = 0.0401$ Δy

$dy = f'(x)\,dx \approx f'(x)\,\Delta x = 2x\,\Delta x \Rightarrow dy = 2(2)(0.01) = 0.04$

Example 2. Suppose that for a certain resistor, resistance varies with temperature as $R = 3.5 + 0.002T^2$ ohms. If T is measured to be 100 degrees celsius, with a possible error of ± 0.1 degrees celsius, what is the approximate maximum error in R?

$\Delta R \approx dR = R'(T)\,\Delta T$

$\Delta T = \pm 0.1$

$R'(T) = 0.004\,T \Rightarrow R'(100) = 0.004\,(100)$

$= 0.4$

$\Delta R \approx (0.4)(\pm 0.1) \Rightarrow \Delta R \approx \pm 0.04$ ohms

Relative error. The quantity $\dfrac{dy}{y}$ is called relative error.

Example 3. In example 2, what is the approximate relative error?

$$\left.\frac{\Delta R}{R}\right|_{T=100} = \frac{\pm 0.04}{R(100)}$$

$$R(T) = 3.5 + 0.002\,T^2$$

$$R(100) = 3.5 + 0.002\,(100)^2 = 3.5 + 20 = 23.5 \text{ ohms.}$$

$$\text{relative error} = \frac{\pm 0.04}{23.5} \approx \pm 0.017 = 1.7\,\%$$

Example 4. A protective coat of thickness 0.5 mm is applied evenly to the surface of a metal sphere of radius 20 cm. Find the approximate number of cubic centimeters of coating used.

$\downarrow$

ΔV

$$R = 20 \text{ cm}, \quad \Delta R = 0.5 \text{ mm} = 0.05 \text{ cm}$$

$$V = \frac{4\pi}{3} R^3$$

$$\Delta V \approx dV = V'(R)\,\Delta R$$

$$= \frac{4\pi}{3}(3R^2)\,\Delta R$$

$$= 4\pi R^2\,\Delta R$$

$$\Rightarrow \Delta V = 4\pi(20)^2(0.05) \text{ cm}^3$$

$$= 4\pi(20)(1.00) \text{ cm}^3$$

$$= 80\pi \text{ cm}^3 = 251.2 \text{ cm}^3$$