Antiderivatives

Given a function $f(x)$, its antiderivative is a function $F(x)$ such that $F'(x) = f(x)$.

The antiderivatives have the following properties:

1. The antiderivatives of x^n ($n \neq -1$) are $\dfrac{x^{n+1}}{n+1} + c$ where c is an arbitrary constant.
2. If an antiderivative of $f(x)$ is $F(x)$ then the antiderivatives of $k\,f(x)$ are $k\,F(x) + c$ where c is some arbitrary constant.
3. If some antiderivatives of $f(x)$ and $g(x)$ are $F(x)$ and $G(x)$ respectively, then the antiderivatives of $f(x)+g(x)$ are $F(x)+G(x)+c$, with c being an arbitrary constant.

Example 1. Find the antiderivatives of $f(x) = 3x^4 + x + 2$.

$$F(x) = 3\,\frac{x^{4+1}}{4+1} + \frac{x^{1+1}}{1+1} + 2\,\frac{x^{0+1}}{0+1} + C$$

$$= 3\,\frac{x^5}{5} + \frac{x^2}{2} + 2x + C$$

Example 2. Find the antiderivatives of $f(x) = 2x^2 + x^3$.

$$F(x) = 2\,\frac{x^{2+1}}{2+1} + \frac{x^{3+1}}{3+1} + C$$

$$= 2\,\frac{x^3}{3} + \frac{x^4}{4} + C$$

$$= \frac{2}{3}x^3 + \frac{1}{4}x^4 + C$$

Example 3. Find the antiderivatives of $f(x) = \sqrt{x} - \dfrac{2}{x^2} - 6$.

$$f(x) = x^{1/2} - 2x^{-2} - 6x^0$$

$$F(x) = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 2\frac{x^{-2+1}}{-2+1} - 6\frac{x^{0+1}}{0+1} + C$$

$$= \frac{x^{3/2}}{3/2} - 2\frac{x^{-1}}{-1} - 6x + C$$

$$= \frac{2}{3}x^{3/2} + 2x^{-1} - 6x + C = \frac{2}{3}x^{3/2} + \frac{2}{x} - 6x + C$$

Example 4. Find the antiderivatives of $g(x) = x^2\sqrt{x} - \dfrac{1}{\sqrt[3]{x}}$.

$$g(x) = x^{2+\frac{1}{2}} - \frac{1}{x^{1/3}} = x^{5/2} - x^{-1/3}$$

$$G(x) = \frac{x^{5/2+1}}{5/2+1} - \frac{x^{-1/3+1}}{-\frac{1}{3}+1} + C$$

$$\Rightarrow G(x) = \frac{x^{7/2}}{7/2} - \frac{x^{2/3}}{2/3} + C$$

$$= \frac{2}{7}x^{7/2} - \frac{3}{2}x^{2/3} + C$$