Fundamental Theorem of Calculus: If f is continuous on the interval $[a, b]$ then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

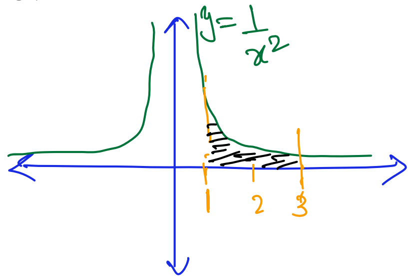
where F is an antiderivative of f.

Example 1. Evaluate $\int_0^3 x^2\,dx$.

$$\int_0^3 x^2\,dx = \frac{x^{2+1}}{2+1}\Big|_0^3 = \frac{x^3}{3}\Big|_0^3$$

$$= \frac{(3)^3}{3} - \frac{0^3}{3} = \frac{27}{3} - 0 = \frac{27}{3} = 9$$

Example 2. Find the area under the curve $y = 1/x^2$ between the lines $x = 1$ and $x = 3$.



$$\text{Area} = \int_1^3 \frac{1}{x^2}\,dx$$

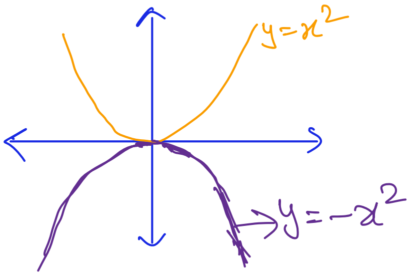
$$= \int_1^3 x^{-2}\,dx$$

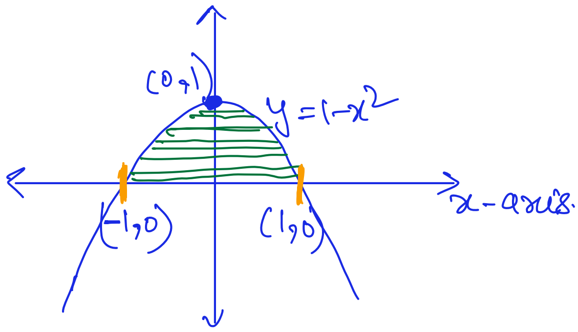
$$= \frac{x^{-2+1}}{-2+1}\Big|_1^3 = \frac{x^{-1}}{-1}\Big|_1^3$$

$$= \left(\frac{3^{-1}}{-1}\right) - \left(\frac{1^{-1}}{-1}\right) = \left(-\frac{1}{3}\right) - (-1)$$

$$= -\frac{1}{3} + 1 = 1 - \frac{1}{3} = \frac{2}{3}$$

Example 3. Find the area of the region bounded by $y = 1 - x^2$ and the x-axis.





$y = 1-x^2$

$y' = -2x$

$y' = 0 \Rightarrow -2x = 0$

$\Rightarrow x = 0$

+ | − sign of $y' = -2x$

0 pt. of max.

$y'' = -2 < 0$ always.

Concave down.

$y = 0$ on x-axis

$1 - x^2 = 0$

$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$$A = \int_{-1}^{1} (1-x^2)\, dx$$

$f(x) = 1 - x^2$

$$F(x) = \frac{x^{0+1}}{0+1} - \frac{x^{2+1}}{2+1}$$

$$= \frac{x}{1} - \frac{x^3}{3}$$

$$A = \int_{-1}^{1} (1-x^2)\, dx = F(1) - F(-1)$$

$$= \left(\frac{1}{1} - \frac{1^3}{3}\right) - \left(\frac{-1}{1} - \frac{(-1)^3}{3}\right)$$

$$= \left(1 - \frac{1}{3}\right) - \left(-1 - \left(\frac{-1}{3}\right)\right)$$

$$= \frac{2}{3} - \left(-1 + \frac{1}{3}\right)$$

$$= \frac{2}{3} - \left(\frac{-2}{3}\right) = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$