Inverse Trigonometric Functions

$$\frac{d}{dx}(\arcsin u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad (|u| < 1),$$

$$\frac{d}{dx}(\arccos u) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad (|u| < 1),$$

$$\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2}\frac{du}{dx}.$$

Example 1. Differentiate $y = \arcsin 2x^3$ with respect to x.

$$y = \arcsin(\underbrace{2x^3}_{u})$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-(2x^3)^2}} \cdot \frac{d}{dx}(2x^3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{6x^2}{\sqrt{1-4x^6}}$$

Example 2. Differentiate $v = (\arctan t)^2$ with respect to t.

$$v = (\underbrace{\arctan t}_{z})^2 \quad \text{(use chain rule)}$$

$$\Rightarrow \frac{dv}{dt} = \frac{d}{dt}(z^2) = \frac{d}{dz}(z^2)\frac{dz}{dt}$$

$$= 2z\frac{dz}{dt}$$

$$= 2(\arctan t)\frac{d}{dt}(\arctan t)$$

Substituting z back

$$\Rightarrow \frac{dv}{dt} = 2(\arctan t) \cdot \frac{1}{1+t^2} = \frac{2(\arctan t)}{1+t^2}$$

Example 3. Differentiate $y = \dfrac{\arccos 2x}{x}$ with respect to x.

a quotient $\Rightarrow$ use quotient rule

$$y = \frac{\arccos 2x}{x} = \frac{u}{v}$$

$$\Rightarrow y' = \frac{u'v - uv'}{v^2}$$

$$u = \arccos 2x \Rightarrow u' = \frac{-1}{\sqrt{1-(2x)^2}} \cdot \frac{d}{dx}(2x) = \frac{-2}{\sqrt{1-4x^2}}$$

$$v = x \Rightarrow v' = 1$$

$$y' = \frac{\frac{-2}{\sqrt{1-4x^2}} \cdot x - \arccos(2x)}{x^2}$$

$$= \frac{-2x - \left(\sqrt{1-4x^2}\right)\arccos(2x)}{x^2\sqrt{1-4x^2}}$$