Derivatives of exponential functions

$$\frac{d}{dx}(b^u) = b^u(\ln b)\,\frac{du}{dx}\,,$$
$$\frac{d}{dx}(e^u) = e^u\,\frac{du}{dx}\,.$$

In particular, $(e^x)' = e^x$ and $(b^x)' = b^x \ln b$.

Example 1. Differentiate $y = 2^{x^2}$ with respect to x .

$y' = 2^{x^2}(\ln 2) \cdot \frac{d}{dx}(x^2)$ [chain rule]

$= 2^{x^2}(\ln 2) \cdot 2x$

$= 2x(\ln 2) \cdot 2^{x^2}$

Example 2. Differentiate $y = e^{\sin x}$ with respect to x.

$y' = e^{\sin x} \cdot (\sin x)'$ [chain rule]

$= \cos x \cdot e^{\sin x}$

Example 3. Differentiate $y = \dfrac{e^x}{e^{\sin x}}$ with respect to x.

$$y = \frac{e^x}{e^{\sin x}} = e^{x - \sin x}$$ [use properties of exp. fns. to simplify]

$$\Rightarrow y' = e^{x - \sin x} \cdot (x - \sin x)'$$ [chain rule]

$$= e^{x - \sin x} \cdot (1 - \cos x)$$

$$= (1 - \cos x)\, e^{x - \sin x}$$

Example 4. Differentiate $y = x\,e^x$.

use product rule.

$$y' = [x]' \cdot e^x + x \cdot [e^x]'$$

$$= e^x + x\,e^x$$

$$= e^x (1 + x)$$

Example 5. Differentiate $y = \dfrac{e^{\cos x} \cdot e^{\arcsin x}}{e^{\arctan x}}$.

First use properties of exp. fns. to simplify y.

$$\Rightarrow y = \frac{e^{\cos x + \arcsin x}}{e^{\arctan x}}$$

$$= e^{\cos x + \arcsin x - \arctan x}$$

$$\Rightarrow y' = e^{\cos x + \arcsin x - \arctan x} \cdot \left[\cos x + \arcsin x - \arctan x\right]'$$

(Chain rule)

$$\Rightarrow y' = \left(-\sin x + \frac{1}{\sqrt{1-x^2}} - \frac{1}{1+x^2}\right) e^{\cos x + \arcsin x - \arctan x}$$

Example 6. Differentiate implicitly to find dy/dx if $e^y = x$.

Diff. both sides w.r.t. x :-

$$\Rightarrow \frac{d}{dx}\left(e^y\right) = \frac{d}{dx}(x)$$

$$\Rightarrow e^y \cdot \frac{dy}{dx} = 1 \quad \text{[Chain rule is used on LHS]}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{e^y}$$

(because $e^y = x$)

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$$