Name: [1 pt]

Problem 1. Find the derivative of $f(x) = (\sec x)^{\tan x}$. [5 pts]

Take ln on both sides:

$$\Rightarrow \ln f(x) = \ln (\sec x)^{\tan x} = \tan x \ln(\sec x)$$

Now diff. both sides:

$$\Rightarrow \frac{1}{f(x)} \cdot f'(x) = [\tan x]' \ln(\sec x) + \tan x [\ln(\sec x)]'$$

(Product rule)

$$\Rightarrow \frac{f'(x)}{f(x)} = \sec^2 x \ln(\sec x) + \tan x \cdot \frac{1}{\sec x} \cdot (\sec x)'$$

$$= \sec^2 x \ln(\sec x) + \tan x \cdot \frac{1}{\sec x} \cdot \sec x \tan x$$

$$= \sec^2 x \ln(\sec x) + \tan^2 x$$

$$\Rightarrow f'(x) = (\sec x)^{\tan x} [\sec^2 x \ln(\sec x) + \tan^2 x]$$

Problem 2. Find all the anitderivatives of $y = \frac{x}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$. [5 pts]

$$y = \sqrt{x} + \frac{1}{x\sqrt{x}} = x^{1/2} + x^{-3/2}$$

$$\Rightarrow \text{Antiderivatives} = \frac{x^{1/2+1}}{\frac{1}{2}+1} + \frac{x^{-3/2+1}}{\frac{-3}{2}+1} + C$$

$$= \frac{x^{3/2}}{3/2} + \frac{x^{-1/2}}{-1/2} + C$$

$$= \frac{2}{3} x^{3/2} - 2x^{-1/2} + C$$

$$= \frac{2}{3} x\sqrt{x} - \frac{2}{\sqrt{x}} + C$$