Final Exam Review

Math I-110 — Final Exam Comprehensive Review (Student Version)

Instructions: Show all work on separate paper. Use exact values unless otherwise directed.

Section 1 — Basic Algebra & Functions

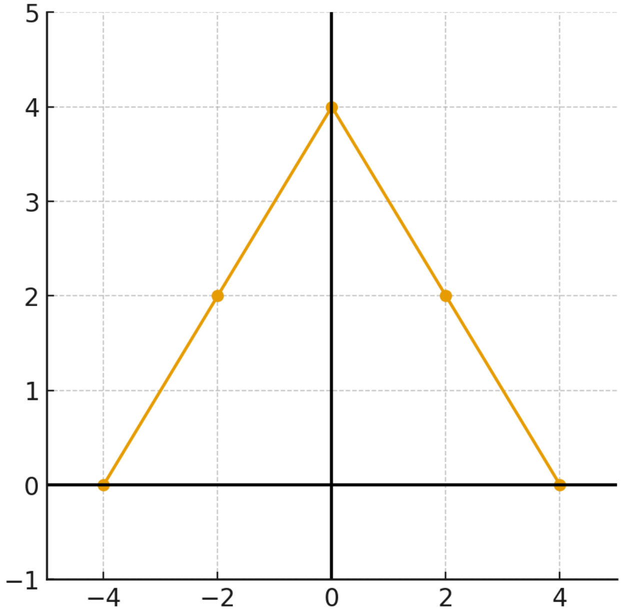
1. Evaluate the expression: $20 - (b - a)^2$ for $a = -3$ and $b = 6$.
2. Translate the phrase into an algebraic expression: "12 less than three times a number."
3. Find the absolute value: $|-8.45|$.
4. Simplify: $\frac{3}{4} - \left(-\frac{5}{6}\right)$.
5. Divide: $-\frac{10}{7} \div \frac{2}{5}$.
6. Evaluate using order of operations: $16 - (4 - 3 \cdot 2^3)$.
7. Simplify: $6(2m - 1) - 5(3m + 4)$.
8. Solve for x: $5x + 7 - 3x = x + 12 - 4x$.
9. Translate into an equation (do **not** solve): A student scored 92, 85, 77, and 88 on tests. What must they score on the next test to have an average of 84?
10. A team played 150 games and had 18 more wins than losses. How many games did they win?
11. Solve for x: $w = \frac{2y - x}{z}$.
12. Simplify using positive exponents only: $\frac{3x^{-4}y^5}{x^2y^{-3}}$.
13. Simplify: $(4x^2)^3$.
14. Plot the points $C(3, -5)$ and $D(-2, 4)$ on a coordinate plane.
15. Graph the line $y = -2x + 3$.
16. For the relation $\{(2, 7), (4, -1), (6, 7), (2, 3)\}$, state the domain and the range, and determine whether it is a function.

NO

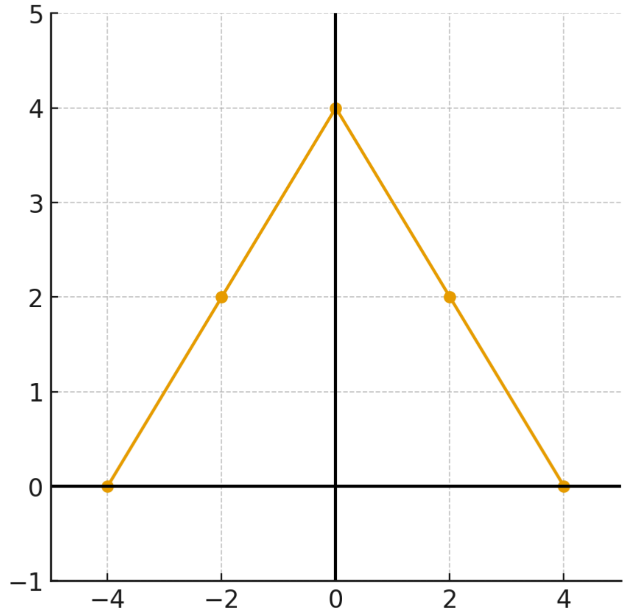
$\{2, 4, 6\}$
Domain

$\{7, -1, 3\}$
Range

17. From the graph of a function f, list all input values where $f(x) = 2$.



18. From the graph of a function f, find $f(-1)$.



$(-\infty,-8)\cup(-8,\infty)$

-8

19. Find the domain of $f(x) = \dfrac{5}{x+8}$ in interval notation.

$x+8 \neq 0 \Rightarrow \boxed{x \neq -8}$

20. Find the domain of $f(x) = \sqrt{12-x}$ in interval notation.

$12-x \geq 0 \Rightarrow (-\infty, 12]$

21. Find the slope of the line through $(4,-2)$ and $(10,7)$.
22. Find the slope of the line through $(-6,1)$ and $(-6,-12)$.
23. Find the slope of the line through $(-8,5)$ and $(1,5)$.
24. Determine the slope and y-intercept of the line $y = -4x + 7.1$.

$m=-4,\ b=7.1$

25. A submarine descends from 200 ft to 1450 ft in 18 minutes. Find the average rate of change in feet per minute.

slope $= \dfrac{1450-200}{18}$

26. Determine whether the lines are parallel, perpendicular, or neither:
   $4x - 3y = 9$
   $3x + 4y = -12.$
27. Determine whether the lines are parallel, perpendicular, or neither:
   $6x + y = 14$

m (x_1, y_1)

$y - y_1 = m(x - x_1)$

$-18x - 3y = 21.$ $y = mx + b$

28. Determine whether the lines are parallel, perpendicular, or neither:

$5x - 10y = 2$

$7x + 14y = -9.$

slopes are equal $\quad$ $m_1 m_2 = -1$

29. Write the equation of the line in point-slope form with slope $\frac{5}{2}$ passing through $(-1, 8)$.
30. Write the equation of the line in point-slope form with slope -7 passing through $(4, -3)$.
31. Find the equation of the line through $(-7, 6)$ and $(-1, 18)$. Write your answer in slope-intercept form.
32. Find the equation of the line through $(2, -9)$ and $(6, -3)$. Write your answer in slope-intercept form.
33. Let $f(x) = 5x - 2$ and $g(x) = x^2 + 4$. Find $f(4) + g(4)$. $\rightarrow$ Find $f(4)$ and $g(4)$, then add
34. Let $f(x) = 3x + 1$ and $g(x) = 2x^2 - 5$. Find $f(6) - g(6)$.
35. Let $f(x) = 8 - 2x$ and $g(x) = x^2 - 1$. Find $f(5) \cdot g(5)$.
36. Let $f(x) = 4x$ and $g(x) = x^2 + 3$. Find $\frac{g(2)}{f(2)}$.
37. Let $f(x) = 3x - 1$ and $g(x) = x + 9$. Find $(f - g)(x)$. $\rightarrow$ $f(x) - g(x)$

$3x - 1 - (x + 9)$

$3x - 1 - x - 9 = 2x - 10$

38. Let $f(x) = x^2 - 3x$ and $g(x) = x - 4$. Find $(f - g)(-5)$.
39. Let $f(x) = 4 - 3x$ and $g(x) = -2x + 7$. Find $(f + g)(x)$.
40. Let $f(x) = 6x^2 - 5$ and $g(x) = 10 - 4x$. Find $(f + g)(x)$.
41. Let $f(x) = x^2 - 16$ and $g(x) = x - 4$. Find $\frac{f(x)}{g(x)}$. $\rightarrow$ $\frac{x^2 - 16}{x - 4} = x + 4$
42. Let $f(x) = 3x - 1$ and $g(x) = 2x + 7$. Find $(f \cdot g)(x)$. $\rightarrow$ $f(x)\,g(x) = (3x - 1)(2x + 7)$

$= 6x^2 + 19x - 7$

Section 2 — Systems, Matrices, Inequalities & Applications

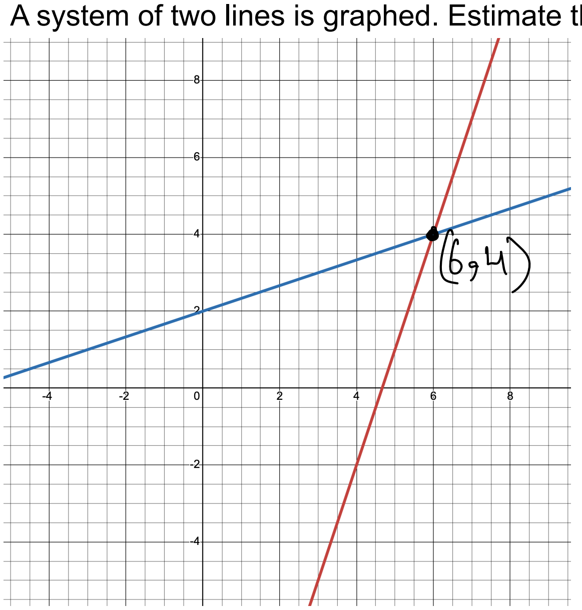
43. Determine whether $(3, -1)$ is a solution to the system:

$x - 2y = 5$ $\rightarrow$ $3 - 2(-1) = 5 \Rightarrow 3 + 2 = 5 \Rightarrow$ True

$y = 4x + 1.$ $\rightarrow$ $-1 = 4(3) + 1 \Rightarrow -1 = 12 + 1 \Rightarrow$ False

$\Rightarrow$ Not a solution.

44. A system of two lines is graphed. Estimate the solution as an ordered pair (x, y) from the graph.



$x + y = 42$

$3.40x + 5.10y = 178.20$

45. Write a system of equations (do not solve): A farmer sells 42 pounds of two nut mixes. One costs $\$3.40$ per pound and the other costs $\$5.10$ per pound. The total revenue is $\$178.20$. Let x be pounds of the \$3.40 mix and y be pounds of the \$5.10 mix.

NO solns

46. When solving a system by substitution, you obtain the equation $0 = -9$. What does this tell you about the system?

$\Rightarrow$ System is inconsistent.

47. Solve the system using substitution:

$y = 5x + 4$
$2x + 3y = 19$.

Substitute or replace y in 2nd eqn. $\rightarrow$ $2x + 3(5x+4) = 19$
$\Rightarrow 17x + 12 = 19 \Rightarrow \boxed{x = \frac{7}{17}}$
$y = 5\left(\frac{7}{17}\right) + 4 = \frac{103}{17}$
$\left(\frac{7}{17}, \frac{103}{17}\right)$

48. Solve the system using elimination:

$4x - y = 9$
$4x + y = 7$.

Add the two eqns $\Rightarrow 8x = 9+7$
$\Rightarrow x = 2$
$\Rightarrow y = 7 - 4(2)$
$\Rightarrow y = -1 \Rightarrow (2, -1)$

49. Solve the system:

$3x + y = 22$
$x - 2y = 1$

Write your solution as an ordered pair, then give the sum of the coordinates.

50. Boxes of lemons sell for $\$7$ each and boxes of oranges (y) sell for $\$11$ each. A total of 620 boxes are sold, and the total revenue is $\$5740$. How many boxes of lemons were sold? (x)

$x + y = 620$
$7x + 11y = 5740$
$\Rightarrow x = 270$

51. State the dimensions of the matrix

2×3

$$\begin{bmatrix} 2 & 1 & -4 \\ 5 & 0 & 3 \end{bmatrix}.$$

A+B or A−B can be computed if A and B have same dimensions

52. Given

$$A = \begin{bmatrix} 4 & -1 \\ 2 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 5 \\ 7 & -8 \end{bmatrix},$$

2×2, 2×2

$A+B = \begin{bmatrix} 1 & 4 \\ 9 & -2 \end{bmatrix}$

$\Rightarrow A - B = \begin{bmatrix} 4-(-3) & -1-5 \\ 2-7 & 6-(-8) \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ -5 & 14 \end{bmatrix}$

compute $A - B$.

53. Find the equilibrium price p for the demand and supply functions

$D(p) = S(p)$

$$D(p) = 5000 - 8p, \quad S(p) = 2400 + 12p.$$

$\Rightarrow 5000 - 8p = 2400 + 12p \Rightarrow p = 130$

54. Given cost and revenue

$P(x) = R(x) - C(x)$
$= 70x - 40000$

$$C(x) = 180x + 40{,}000, \quad R(x) = 250x,$$

find the profit function $P(x)$.

$P(x) = R(x) - C(x)$. $R(x) = 525x$, $C(x) = 30000 + 350x$

55. A company has fixed costs of $\$30{,}000$, variable costs of $\$350$ per unit, and revenue of $\$525$ per unit. Find the profit or loss if 600 units are produced and sold. Profit of 75000

$\Rightarrow P(x) = 525x - 350x - 30000$
$= 175x - 30000$

56. Solve for n:

Solution set = All real numbers or $(-\infty, \infty)$ $\Leftarrow$

$$8n - (3n - 12) \geq 4 + 5n.$$

$\Rightarrow 8n - 3n + 12 \geq 4 + 5n$
$\Rightarrow 5n + 12 \geq 4 + 5n$
$\Rightarrow 8 \geq 0 \rightarrow$ True always

Describe the solution set.

$x < 9 \Rightarrow$ 9

57. Graph the inequality $x \leq 9$ on a number line and write the solution set in set-builder notation and interval notation.

9

Interval $= (-\infty, 9]$

set-builder $= \{x : x \leq 9\}$

58. Solve and give the answer in interval notation:

$\left(\frac{5}{2}, \infty\right) \Leftarrow$

$$7(3x - 2) > 5x + 26.$$

59. Translate into an inequality: "A number is no more than 11.8." $\Rightarrow x \leq 11.8$

$2\times2 \qquad\qquad 2\times2 \qquad\qquad 2\times1$

$$A = \begin{bmatrix} 4 & -1 \\ 2 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 5 \\ 7 & -8 \end{bmatrix}, \quad C = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$

CA = Not Possible

$4 \leftrightarrow -3$
$+$
$-1 \leftrightarrow 7$

$2 \leftrightarrow -3$
$+$
$6 \leftrightarrow 7$

AC = [first row of A with first Column of C — same dimension
$4(-3) + (-1)\times 7$
$2(-3) + 6(7)$]

$(2\times2)(2\times1) = (2\times1)$

$$= \begin{bmatrix} 4(-3)+(-1)(7) \\ 2(-3)+6(7) \end{bmatrix} = \begin{bmatrix} -12-7 \\ -6+14 \end{bmatrix} = \begin{bmatrix} -19 \\ 8 \end{bmatrix}$$

(*) multiplying a row with a column.

$$\underset{1\times2}{\begin{bmatrix} 1 & 2 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} 3 \\ 4 \end{bmatrix}} = \begin{bmatrix} 1(3)+2(4) \end{bmatrix} = [11]$$

answer $\rightarrow 1\times1$

$$AB = \begin{bmatrix} A_{row1} \cdot B_{col1} & A_{row1} \cdot B_{col2} \\ A_{row2} \cdot B_{col1} & A_{row2} \cdot B_{col2} \end{bmatrix} = \begin{bmatrix} -19 & 28 \\ 36 & -38 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 \\ 2 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 5 \\ 7 & -8 \end{bmatrix}.$$

(*) Find domain of $\dfrac{3}{\sqrt{2x-6}}$. $\Rightarrow$ $(3, \infty)$

For $\sqrt{2x-6}$ to be well-defined, $2x-6 \geq 0$ $\Rightarrow 2x-6 > 0$

cannot have equality

$\sqrt{2x-6} = 0 \Rightarrow 2x-6 = 0$

should not happen

$\Rightarrow 2x > 6$

$\Rightarrow x > 3$

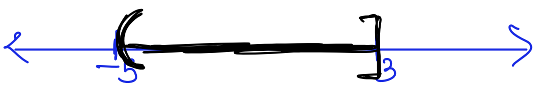
60. A chemical reaction must not exceed $102.4^\circ \mathrm{C}$. Convert this temperature to degrees Fahrenheit using

$$F = \frac{9}{5}\overset{102.4}{C} + 32.$$

$\Rightarrow F = \frac{9}{5}(102.4) + 32$

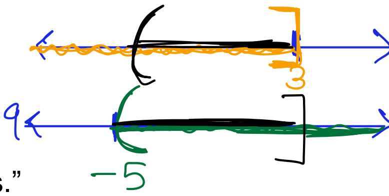
$= 216.32$

61. Solve and graph the compound inequality:



$$-9 < 2t + 1 \le 7.$$

$\Rightarrow$ $2t+1 \le 7$ and $2t+1 > -9$



$(-5, 3]$

62. Write the domain of $f(x) = \sqrt{x+12}$ in interval notation. $[-12, \infty)$

63. Translate the phrase to an expression: "10 decreased by the product of two numbers." $10 - xy$

64. Evaluate $\dfrac{12a}{4b}$ when $a = 3$ and $b = 2$. $= \frac{9}{2}$

65. Simplify: $3(2x - 7) - 4(x - 5)$. $\Rightarrow$ $2x - 1$

66. Solve for r: $5r - (2r - 9) = 40$. $\Rightarrow r = 31/3$

67. Find the slope of the line through $(14, -3)$ and $(-2, 9)$.

(x_1, y_1) (x_2, y_2) $\Rightarrow m = \dfrac{-3-9}{14-(-2)} = \dfrac{-12}{16} = \dfrac{-3}{4}$

Section 3 — Polynomials, Operations, & Factoring

68. Determine the degree of the polynomial $= 8$

$$\underbrace{7x^4y^2}_{6} - \underbrace{12x^3y^5}_{8} + \underbrace{2xy}_{2}.$$

69. Write in standard form and state the degree:

$5''$

$$4p^2 - 3p^5 + 6p^3 - 9. = -3p^5 + 6p^3 + 4p^2 - 9$$

70. Add the polynomials:

$= 13m^4 + 5m^3 - 8m + 6$

$$(5m^4 + 7m^3 - 12m + 9) + (8m^4 - 2m^3 + 4m - 3).$$

71. The height of a rocket is given by $h(t) = 7t^2 + 9t$, where t is in seconds and h in meters. Find how far it travels after 6 seconds.

$\Rightarrow h(6) = 7(6)^2 + 9(6) = 306$ m

72. Evaluate $f(x) = 4x^3 - 5x^2 + 6x - 4$ at $x = -3$. $\Rightarrow f(-3) = 4(-3)^3 - 5(-3)^2 + 6(-3) - 4 = -175$

73. Subtract the polynomials:

$= 4x^5 - 12x^3 + 11x - 9$

$$(6x^5 - 3x^3 + 7x - 8) - (2x^5 + 9x^3 - 4x + 1).$$

74. Perform the indicated operations:

$= 13y$

$$(3y^2 + 5y - 10) + (-2y^2 + 7y + 4) - (y^2 - y - 6).$$

75. Multiply the monomials:

$= -15a^6b^4$

$$(-3a^4b^3)(5a^2b).$$

76. Find the product:

$$x^3(4x^2 - 3x + 8). = 4x^5 - 3x^4 + 8x^3$$

77. Multiply:

$$(x+2)(x^2-5x+6).$$

$= x(x^2-5x+6)+2(x^2-5x+6)$

$= x^3-5x^2+6x+2x^2-10x+12$

$= x^3-3x^2-4x+12$

78. Multiply:

$$(3x-4y)(2x+y).$$

$= 3x(2x+y)-4y(2x+y)$

$= 6x^2+3xy-8xy-4y^2$

$= 6x^2-5xy-4y^2$

79. Simplify:

$$(x+5)(x-5)-(x-3)^2.$$

$= x^2-5^2-[x^2-2(x)(3)+3^2]$

$= x^2-25-(x^2-6x+9)$

$= \cancel{x^2}-25-\cancel{x^2}+6x-9$

$= 6x-34$

80. Given $f(x)=x^2+2x-1$, simplify

$$f(a+h)-f(a).$$

$f(a+h) = (a+h)^2+2(a+h)-1$

$= a^2+2ah+h^2+2a+2h-1$

$= \cancel{a^2}+2ah+h^2+\cancel{2a}+2h\cancel{-1}\cancel{-a^2}\cancel{-2a}\cancel{+1}$

$= h^2+2ah+2h$

81. Factor completely:

$$x^3-6x^2+9x-54.$$

$= x^2(x-6)+9(x-6)$

$= (x-6)(x^2+9)$

82. Factor out the greatest common factor:

$$12x^4-30x^3.$$

$= 6x^3(2x-5)$

83. Factor completely:

$$q^2-49.$$ $= q^2-7^2 = (q+7)(q-7)$

84. Factor out the greatest common factor:

$$-20r^3+15r^2-5r.$$ $= -5r(4r^2-3r+1)$

85. Factor completely:

1×40
2×20
4×10
5×8

$$x^2-13x+40.$$

$= x^2-5x-8x+40$

$= x(x-5)-8(x-5)$

$= (x-5)(x-8)$

$-20 = -1\times 20$
$= -2\times 10 = -4\times 5$

86. Factor completely:

$$a^3+a^2-20a.$$

$= a(a^2+a-20)$

$a[a^2-4a+5a-20]$

$= a[a(a-4)+5(a-4)] = a(a-4)(a+5)$

87. Write as a product of prime polynomials:

$$18x^2+33x+15.$$

$= 3(6x^2+11x+5)$

$= 3(6x^2+6x+5x+5)$

$= 3[6x(x+1)+5(x+1)]$

$= 3(x+1)(6x+5)$

Section 4 — Radicals, Exponents, & Quadratics

$\boxed{\sqrt[3]{x^3}=x}$

88. Find all square roots of 1764. $\Longrightarrow$ $42, -42$

89. Simplify (assume variables are nonnegative): $\sqrt{b^{10}}$. $= b^5$

90. Simplify: $\sqrt[3]{-\dfrac{54}{125}}$.

$= \sqrt[3]{\dfrac{(-1)^3\, 3^3\cdot 2}{5^3}} = \dfrac{\sqrt[3]{(-1)^3}\,\sqrt[3]{3^3}\,\sqrt[3]{2}}{\sqrt[3]{5^3}} = \dfrac{-3}{5}\sqrt[3]{2}$

91. Write $k^{4/3}$ using radical notation. $= k^{4/3} = k^{4\cdot\frac{1}{3}} = (k^4)^{1/3} = \sqrt[3]{k^4}$

92. Write $\sqrt[5]{x^7}$ using exponential notation. $= x^{7/5}$

93. Simplify: $(3r^{-2}s^3)^4$. $= 3^4(r^{-2})^4(s^3)^4 = 81r^{-8}s^{12} = \frac{81s^{12}}{r^8}$

94. Simplify by factoring inside the radical: $\sqrt{75x^9y^{12}}$. $= \sqrt{25\cdot 3\, x^8\cdot x\, y^{12}} = \sqrt{5^2\cdot 3\cdot (x^4)^2\cdot x\,(y^6)^2} = 5x^4y^6\sqrt{3x}$

95. Simplify: $\sqrt{\dfrac{16x^4y}{49}}$. $= \dfrac{4x^2\sqrt{y}}{7}$

96. Rationalize the denominator: $\dfrac{5}{\sqrt{6}}$. $= \dfrac{5}{\sqrt6}\cdot\dfrac{\sqrt6}{\sqrt6} = \dfrac{5\sqrt6}{6}$

97. Find a polynomial $f(x)$ with zeros at $x=-3$, $x=2$, and $x=5$, and with $f(1)=-24$.

98. A ball is thrown upward with initial velocity so that $h(t)=-16t^2+96t$, where h is height (feet) and t is time (seconds). How many seconds until it hits the ground? t for which $h(t)=0$

$\Rightarrow -16t^2+96t=0$

$-16t(t-6)=0$

$\Rightarrow t=0$ or $t=6$

99. Solve and find the sum of the roots:

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ 7 and 2

$$x^2-9x+14=0.$$

100. Find the x-intercepts of $f(x)=4x^2-28x+36$. value of x for which $f(x)=0$

$4x^2-28x+36=0$

$4(x^2-7x+9)=0$

$\Rightarrow x^2-7x+9=0$

$a=1,\ b=-7,\ c=9$

$\Rightarrow x = \dfrac{-(-7)\pm\sqrt{49-36}}{2(1)}$

$\Rightarrow x = \dfrac{7\pm\sqrt{13}}{2}$

101. Let $f(x)=(x+6)^2-25$. Find all x such that $f(x)=31$.

$\Rightarrow (x+6)^2-25=31 \Rightarrow (x+6)^2=31+25=56$

$(x+6)^2=56$

$\Rightarrow x+6=\pm\sqrt{56}$

$\Rightarrow x=-6\pm\sqrt{56}$

$\Rightarrow x=-6\pm2\sqrt{14}$

102. Solve:

$$5x^2-7x-6=0.$$

$x = 2,\ -3/5$

Quadratic formula

103. Sketch the graph of $f(x)=(x-2)^2+3$ and state the vertex.

104. Without graphing, find the vertex of

$$f(x)=\frac{3}{4}(x+8)^2-5.$$

105. Without graphing, find the maximum or minimum value of

$$f(x)=-2(x-1)^2+7$$

and the x-value where it occurs.

106. Write $f(x)=3x^2-12x+4$ in the form $f(x)=a(x-h)^2+k$.

107. The cost of producing x gadgets is

$$C=3x^2-48x+400.$$

How many gadgets should be produced to minimize the cost?

Section 5 — Composition, Inverses, Exponential & Logarithmic Functions

108. Given $f(x)=\sqrt{4x-3}$ and $g(x)=2x+5$, find $(g\circ f)(x)$.

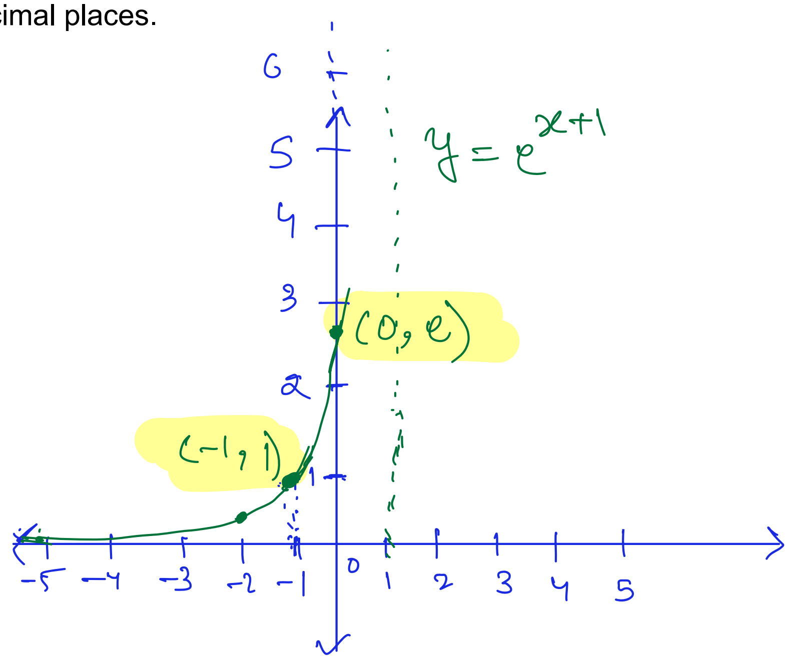
109. Write $h(x)=(3x+4)^2$ in the form $h(x)=f(g(x))$ by choosing appropriate functions f and g.

110. Determine whether $h(x)=4x^3+1$ is one-to-one and find its inverse function.

111. Determine whether $f(x)=7x-9$ is one-to-one and find its inverse function.

112. Graph $y = 3^x - 2$. State the horizontal asymptote and at least two points on the graph.
113. Use $V(t) = 3200(0.91)^t$ to find the value after 12 years.
114. Simplify: $\log_4\left(\frac{1}{64}\right)$.
115. Rewrite $\log_5(25) = x$ in exponential form.
116. Rewrite $3^4 = 81$ in logarithmic form.
117. Solve: $\log_2\left(\frac{1}{32}\right) = x$.
118. Expand: $\log(12x^5)$.
119. Expand: $\log\left((3ab)^4\right)$.
120. Condense: $\ln x + 2\ln y - \ln 5$.
121. Expand: $\ln\left(\frac{B^4}{A^3C}\right)$.
122. Evaluate $\log_9(12)$ and round to four decimal places.
123. Given $\log(3) = 0.477$ and $\log(7) = 0.845$, find $\log_{21}(9)$.
124. Sketch the graph of $f(x) = e^{x+1}$. State the horizontal asymptote and two key points.
125. Solve: $5^{x+1} = 125$.
126. Solve: $10 + 7e^x = 80$.
127. The function $A(t) = 600e^{-0.142t}$ models radioactive decay. Find the time required for the substance to decay to half of its original amount. Round to two decimal places.

x	$f(x)$
-1	$e^{-1+1} = 1$
0	$e^{0+1} = e^1 \approx 2.7$
1	$e^{1+1} = e^2 \approx 7.4$
-2	$e^{-2+1} = e^{-1} \approx 0.4$



Horizontal Asymptote → line to which the graph gets arbitrarily close but never actually intersects it.

Horizontal → Horizontal

Asymptote → straight line

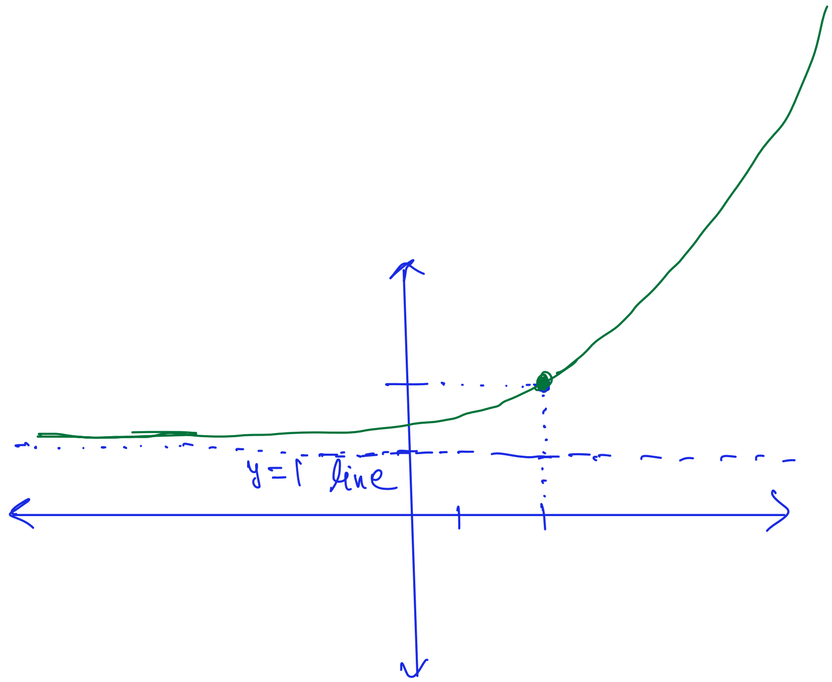
$y = e^{x+1} > 0$ always and never intersects x-axis.

$\Rightarrow$ Horizontal asymptote is $y = 0$

Similar Problems

$$f(x) = \underbrace{e^{x-2}}_{>0} + 1$$

Horizontal Asymptote is $\boxed{y=1}$



126. Solve. $10 + 7e \quad = 80.$

127. The function $A(t) = 600e^{-0.142t}$ models radioactive decay. Find the time required for the substance to decay to half of its original amount. Round to two decimal places.

t for which $A(t) = \frac{1}{2}$ (original amount)

$$A(0) = 600\, e^{-0.142(0)} = 600$$

$$\Rightarrow A(t) = \frac{1}{2}(600) = 300$$

$$\Rightarrow 600\, e^{-0.142t} = 300 \Rightarrow e^{-0.142t} = \frac{300}{600} = \frac{1}{2}$$

$$\Rightarrow e^{-0.142t} = 0.5 \Rightarrow \ln\left(e^{-0.142t}\right) = \ln(0.5)$$

$$\Rightarrow -0.142t\,\underbrace{(\ln e)}_{1} = \ln(0.5)$$

$$\Rightarrow -0.142t = \ln 0.5 \Rightarrow t = \frac{\ln 0.5}{-0.142}$$

$$\Rightarrow t = \boxed{4.88}$$

97. Find a polynomial $f(x)$ with zeros at $x=-3$, $x=2$, and $x=5$, and with $f(1)=-24$.

$$f(x) = \underline{a}\,(x+3)\,(x-2)\,(x-5)$$

3 factors of $f(x)$

$x=-3 \Rightarrow x+3=0$

$x=2 \Rightarrow x-2=0$

$x=5 \Rightarrow x-5=0$

$$\underbrace{f(1)}_{=} = -24$$

$$\Rightarrow a(1+3)(1-2)(1-5) = -24 \Rightarrow a(4)(-1)(-4) = -24$$

$$\Rightarrow 16a = 24 \Rightarrow a = \frac{24}{16} \Rightarrow a = \frac{3}{2}$$

$$\Rightarrow f(x) = \frac{3}{2}(x+3)(x-2)(x-5)$$

$$= \frac{3}{2}\left[(x+3)(x^2-2x-5x+10)\right]$$

$$= \frac{3}{2}\left[(x+3)(x^2-7x+10)\right]$$

$$= \frac{3}{2}\left[x^3-7x^2+10x+3x^2-21x+30\right]$$

$$= \frac{3}{2}\left[x^3-4x^2-11x+30\right]$$

$$= \frac{3}{2}x^3-6x^2-\frac{33}{2}x+45$$

103. Sketch the graph of $f(x) = (x-2)^2 + 3$ and state the vertex. form

104. Without graphing, find the vertex of

$$f(x) = \frac{3}{4}(x+8)^2 - 5.$$

105. Without graphing, find the maximum or minimum value of

$$f(x) = -2(x-1)^2 + 7$$

and the x-value where it occurs.

106. Write $f(x) = 3x^2 - 12x + 4$ in the form $f(x) = a(x-h)^2 + k$.

107. The cost of producing x gadgets is

$$C = 3x^2 - 48x + 400.$$

How many gadgets should be produced to minimize the cost?

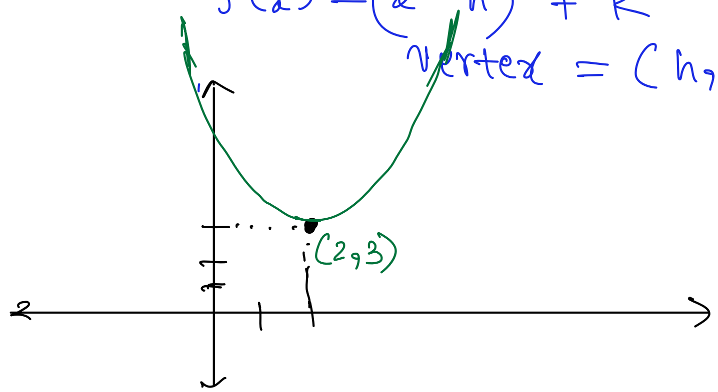
$f(x) = (x-h)^2 + K$

vertex $= (h, K)$

103

$f(x) = (x-2)^2 + 3$

vertex $= (2, 3)$



104

$f(x) = \frac{3}{4}(x+8)^2 - 5$

Vertex $= (-8, -5)$

105

$f(x) = -2(x-1)^2 + 7$ has a maximum.

maximum occurs at vertex. vertex $= (1, 7)$

$x = 1 \Rightarrow$ max. value $= 7$

106

$f(x) = 3x^2 - 12x + 4 \rightsquigarrow f(x) = a(x-h)^2 + K$

$h = \frac{-b}{2a} = \frac{-(-12)}{2(3)} = \frac{12}{6} = 2$, $a = 3$

$K = f(h) = f(2) = 3(2)^2 - 12(2) + 4 = -8$

$\Rightarrow f(x) = 3(x-2)^2 - 8$

107 $C(x) = 3x^2 - 48x + 400$ (Cost function)

$\Rightarrow$ want to minimize cost, x for which $C(x)$ is minimum.

minimum occurs at vertex.

$$x = \frac{-b}{2a} \Rightarrow x = \frac{-(-48)}{2(3)} = \frac{48}{6} = 8$$

8 units should be produced.