1.3 Solving Equations

Objective: Determine whether equations are equivalent

Determine whether the two equations in each pair are equivalent

a. $3x = 21$ and $x + 4 = 11$

$\frac{1}{3} \cdot 3x = \frac{1}{3} \cdot 21$

$x = 7$

$x + 4 = 11$

$-4 \quad -4$

$x = 7$

⇒ Equivalent.

b. $12 - x = 3$ and $2x = 20$

$12 - x = 3$

$-12 \quad -12$

$-x = -9$

$-1 \quad -1$

$x = 9$

$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 20$

$x = 10$

⇒ Not Equivalent

Objective: Solve equations using the addition and multiplication principles

Solve.

a. $3x + 3 = 6$

$3x = 6 - 3 = 3$

$3x = 3$

$x = \frac{3}{3} \Rightarrow x = 1$

b. $5y - 4 = 21$

c. $2(x - 3) = 14$

$x - 3 = \frac{14}{2} = 7$

$x = 7 + 3 \Rightarrow x = 10$

d. $\frac{m}{6} + 5 = 11$

e. $4z + 25 = 9$

f. $9 - 2k = -5$

$-9 \quad -9$

$-2k = -14$

$-2 \quad -2$

$\Rightarrow k = 7$

g. $2y - 8 = 9$

h. $\frac{2}{3}x = 30$

Objective: Simplify expression by combining like terms

Simplify to form an equivalent expression by combining like terms. Use the distributive law as needed.

a. $9x^2 + 9x$

$= 9x^2 + 9x$

b. $16a - a$

$= 15a$

c. $n - 8n$

$= -7n$

d. $5x - 3x = 8x$

$2x = 8x$

$-2x \quad -2x$

$2x - 2x = 8x - 2x$

$0 = 6x \Rightarrow 6x = 0$

e. $18x - 12 + 3x + 8$

f. $-7x^2 + 3x + 5x^3 - x^3 + 3x^2 - x$

$= -4x^2 + 2x + 4x^3$

$= 4x^3 - 4x^2 + 2x$

g. $2x + 3(5x - 7)$

h. $7x - (2x + 5)$

$7x - 2x - 5$

$= 5x - 5$

i. $3y - 7 - (5 - 2y)$

j. $2(x - 3) + 4(7 - x)$

$2x + 2(-3) + 4 \cdot 7 + 4(-x)$

$2x - 6 + 28 - 4x$

$-2x + 22$

k. $-2(x - 5) - [7 - 3(2x - 5)]$

$-2x + (-2) \cdot (-5) - [7 - 3 \cdot (2x) + (-3)(-5)]$

$-2x + 10 - [7 - 6x + 15]$

$-2x + 10 - [22 - 6x]$

$-2x + 10 - 22 + (-1) \cdot (-6x)$

$-2x + 10 - 22 + 6x$

$= 4x - 12$

l. $5 - 2x + 3[2 - 4(5x + 1)]$

Solving Linear Equations

a. $4x + 5x = 63$

b. $3x - 7x = 60$

c. $\frac{1}{4}y - \frac{2}{3}y = 5$

$\left(\frac{1}{4} - \frac{2}{3}\right)y = 5$

$\frac{3-8}{12}y = 5 \Rightarrow \frac{-5}{12}y = 5$

$12 \cdot \frac{-5}{12}y = 12 \cdot 5 \Rightarrow -5y = 60$

$\frac{-5y}{-5} = \frac{60}{-5}$

$\Rightarrow y = -12$

d. $\frac{3}{5}t - \frac{1}{2}t = 3$

e. $4(x - 3) - t = 6$

f. $2(t + 5) + t = 4$

g. $3(x + 4) = 7x$

h. $3(y + 5) = 8y$

$\Rightarrow 3y + 15 = 8y$

$-3y \quad -3y$

$\Rightarrow 15 = 8y - 3y \Rightarrow 15 = 5y$

$\Rightarrow \frac{5y}{5} = \frac{15}{5}$

$\Rightarrow y = 3$

i. $70 = 10(3t - 2)$

j. $27 = 9(5y - 2)$

k. $1.8(2 - n) = 9$

l. $2.1(3 - x) = 8.4$

$2.1(3) - (2.1)x = 8.4$

$6.3 - 2.1x = 8.4$

$-6.3 \quad -6.3$

$\frac{-2.1x}{-2.1} = \frac{2.1}{-2.1}$

$\Rightarrow x = -1$

Types of equations

Contradiction: True for no value of x

Conditional: True for some values of x

Identity: True for every value of x

Objective: Classify equations as conditional, ~~and~~ identity ~~or~~ a contradiction

a. $2x + 2 = 2(x + 1)$

$2x + 2 = 2x + 2$

$-2 \quad -2$

$2x = 2x$

$-2x \quad -2x$

$0 = 0$

Identity

b. $7x - 2 - 3x = 4x$

$4x - 2 = 4x$

$-4x \quad -4x$

$-2 = 0$

Contradiction

c. $x + 2 = x + 3$

d. $3t + 5 + t = 5 + 4t$

e. $2 + 9x = 3(4x + 1) - 1$

f. $3x - (8 - x) = 6x - 2(x + 4)$

g. $4 + 7x = 7(x + 1)$

h. $\frac{1}{3}(2x - 7) = \frac{1}{2}(x + 3)$

i. $-9t + 2 = -9t - 7(6 \div 2(49) + 8)$

j. $2\{9 - 3[-2x - 4]\} = 12x + 42$

$2\{9 - 3(-2x) + (-3)(-4)\} = 12x + 42$

$2\{9 + 6x + 12\} = 12x + 42$

$2\{6x + 21\} = 12x + 42$

$2(6x) + 2(21) = 12x + 42$

$12x + 42 = 12x + 42$

$-12x \qquad -12x$

$42 = 42$ Identity

k. $-9t + 2 = 2 - 9t - 5(8 + 4(1 + 3^4))$

$3^4 = 81$

l. $3\{7 - 2[7x - 4]\} = -40x + 45$