1.4 Introduction to Problem Solving

Five Steps for Problem Solving with Algebra

1. Familiarize yourself with the problem.

2. Translate to mathematical language.

3. Carry *out* some mathematical manipulation.

4. Check your possible answer in the original problem.

5. State the answer clearly.

Example: For the problem, familiarize yourself with the situation. Then translate to mathematical language. Do not actually solve the problem, just carry out the first two steps of the five-step strategy.

The sum of two numbers is 19. One of the numbers is 3 more than the other. What are the numbers?

Let us suppose one of the numbers is x and the other is y.

$y = x + 3$ ← Find y

$x + y = 19$

$x + (x+3) = 19$

$x + x + 3 = 19 \Rightarrow 2x + 3 = 19$ ← Find x

Example: For the problem, familiarize yourself with the situation. Then translate to mathematical language. Do not actually solve the problem, just carry out the first two steps of the five-step strategy.

In her single-person scull, Maja can maintain a speed of 4.8 mph in still water. If she is paddling into a 2.1 -mph current, how long will it take her to complete the 9-mi route?

Let t (hours) be the time taken to complete the 9-mi route.

Her effective speed $= 4.8 - 2.1$

$= 2.7$ mph.

$2.7t = 9$

Example: For the problem, familiarize yourself with the situation. Then translate to mathematical language. Do not actually solve the problem, just carry out the first two steps of the five-step strategy.

The degree measures of the angles in a triangle are three consecutive integers. Find the measures of the angles.

Sum of all the three angle of a triange is 180 degrees

Let one of the angles to be x.
(Smallest one)

Then the other angle are $x+1$, $x+1+1 = x+2$

$x + x+1 + x+2 = 180$ ⟶ Solve for x.

Solve Each Application Problem

Example: The sum of two numbers is 57. If one of the numbers is 6 more than twice the other, what are the numbers?

Example: The faculty discount at a bookstore is 15%. If a sweatshirt after the discount was $32.30. What was the original price of the sweatshirt?

Let the original price be x.

Then Discount $= \frac{15}{100} \cdot x$

$$x - \frac{15}{100}x = 32.30$$

$$x - 0.15x = 32.30 \Rightarrow 0.85x = 32.30$$

$$\Rightarrow x = \frac{32.30}{0.85} = \$38$$

⇒ The original price of the sweatshirt was 38 $.

Example: One angle of a triangle has the same measure as a second angle. The third angle measures 10° more than three times the measure of the first angle. Find the measures of the angles.

Let the first angle be x. (degrees) $\Rightarrow$ second angle is also x. (degrees)

The third angle is $3x+10$.

Sum of all three angles is 180°.

$$x + x + 3x + 10 = 180 \Rightarrow 5x = 180 - 10 = 170$$

$$\Rightarrow x = \frac{170}{5} = 34 \Rightarrow 3x+10 = 3(34)+10 = 102+10 = 112$$

Therefore, the three angles are $34^{\circ}, 34^{\circ}, 112^{\circ}$.

Example: Axis is moving from City A to City B. The average monthly rent of an apartment in City B is \$1610. This is seven-ninths of the average monthly rent of an apartment in City A. What is the average monthly apartment rent in City A?

Let the rent in city A be x.

$\Rightarrow$ rent in city B is $\frac{7}{9}x = 1610$

$$\Rightarrow \frac{7}{9}x = 1610 \Rightarrow 7x = 9 \cdot (1610)$$

$$\Rightarrow x = \frac{9}{7} \times 1610 \; 230 = 2070$$

$\Rightarrow$ The rent in city A is \$2070

Example: Quick Storage prices flash drives by raising the wholesale price 50% and adding $1.50. What must a drive's wholesale price be if it is being sold for $24.00?

Let the wholesale Price be x.

Raise x by 50 % means $\rightarrow x + \frac{50}{100}x$

$\Rightarrow x + \frac{1}{2}x = \frac{3}{2}x$

Selling Price $= \frac{3}{2}x + 1.5 = 24$

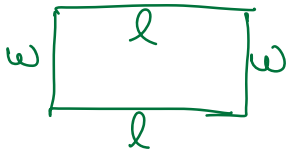
$\Rightarrow \frac{3}{2}x + 1.5 = 24 \Rightarrow \frac{3}{2}x = 24 - 1.5 = 22.5$

$\Rightarrow \frac{3}{2}x = 22.5 \Rightarrow x = \frac{2}{3}(22.5)\ 7.5 = 15$

$\Rightarrow$ The wholesale Price is $\$15$.

Example: The length of a rectangular mirror is twice its width, and its perimeter is 66 cm. Find the length and the width of the mirror.

Perimeter $= l + l + w + w = 2(l + w)$



Let width to be x.

Then length would be $2x$

Perimeter $= 2(2x + x) = 66$

$\Rightarrow 2(3x) = 66 \Rightarrow 6x = 66$

$\Rightarrow x = \frac{66}{6} = 11 \Rightarrow x = 11$

$\Rightarrow$ The width is 11 Cm.

$\Rightarrow$ length is 22 Cm.