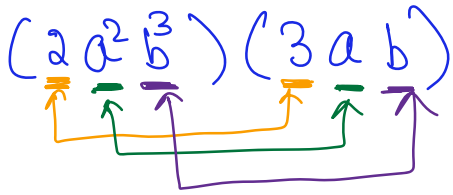
1.6 Notes: Properties of Exponents

The Product Rule: $a^m \cdot a^n = a^{m+n}$

The Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

Example: Multiply and simplify $(2a^2b^3)(3ab)$



$= 6(a^2 \cdot a^1)(b^3 \cdot b^1)$

$= 6a^3b^4$

Example: Divide and simplify $\frac{-8x^7y^5}{2x^2y^3}$

$\frac{-8x^7y^5}{2x^2y^3} = -\frac{8}{2}\left(\frac{x^7}{x^2}\right)\left(\frac{y^5}{y^3}\right) = -4x^5y^2$

Example: simplify

a. x^5x^{12}

$= x^{17}$

b. $(-5x^5y^{10})(4x^3y^7)$

$= -20x^{5+3}y^{10+7}$

$= -20x^8y^{17}$

c. $a^4 * a^8$

$= a^{12}$

d. $(-7a^{10}b^2)(2ab^3)$

$= -14a^{11}b^5$

Example: Simplify

a. $\frac{m^{11}}{m^7}$

$= m^4$

b. $\frac{-24m^6n^8}{-8m^5n^2}$

$= 3mn^6$

c. $\frac{m^{15}}{m^5}$

$= m^{10}$

d. $\frac{-18x^{10}y^2}{2x^7y}$

$= -9x^3y$

The Zero Product

For any nonzero real number: $\rightarrow$ $a^0 = 1$

$(a \neq 0)$

Example: Evaluate

a. x^0 for x=3

$3^0 = 1$

b. $-x^0$ for x=3

$-3^0 = -1$

c. $(-x)^0$ for x=3

$(-3)^0 = 1$

Example: Evaluate

a. x^0 for x=-2

$(-2)^0 = 1$

b. $-x^0$ for x=-2

$-(-2)^0 = -1$

c. $(-x)^0$ for x=-2

$(-(-2))^0$

$= 2^0 = 1$

d. $(5x)^0$ for x=-2

$(5(-2))^0$

$= (-10)^0 = 1$

e. $5x^0$ for x=-2

$5(-2)^0$

$= 5$

Negative Integers as exponents

For any non-zero real number a and any integer n: $a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$

$(a \neq 0)$

Example: Express each of the following without negative exponents and, if possible, simplify

a. $6x^{-3}y^4$

$= 6 \cdot \frac{1}{x^3} \cdot y^4$

$= \frac{6y^4}{x^3}$

b. $\frac{ab^{-3}c^{-5}}{x^{-4}y^{-6}}$

$= \frac{ax^4y^6}{b^3c^5}$

c. $7^{-3}7^6$

$= 7^{-3+6}$

$= 7^3$

d. $\frac{x^{-4}}{x^{-2}}$

$= x^{-4-(-2)}$

$= x^{-2} = \frac{1}{x^2}$

Example: Express each of the following without negative exponents and, if possible, simplify

a. 5^{-2}

$= \frac{1}{5^2}$

$= \frac{1}{25}$

b. -5^{-2}

$= -\frac{1}{5^2}$

$= \frac{-1}{25}$

c. $(-5)^{-2}$

$= \frac{1}{(-5)^2}$

$= \frac{1}{25}$

d. $\frac{1}{5^{-2}}$

$= 5^2$

$= 25$

Write an equivalent expression without negative exponents.

a. $\frac{7a^2b^{-4}}{c^6d^{-10}}$

$= \frac{7a^2d^{10}}{b^4c^6}$

b. $\frac{5a^{-4}b^{-3}}{c^{-5}d^6}$

$= \frac{5c^5}{a^4b^3d^6}$

Simplify and write the answer using positive exponents.

a. $(4x^{-1}y^3z^{-7})(2xyz^2)$

$= 8x^{-1+1}y^{3+1}z^{-7+2}$

$= 8x^0y^4z^{-5}$

$= \frac{8y^4}{}$

b. $(3x^4y^{-2}z^6)(3x^{-2}y^{-5}z)$

z^5

Simplify and write and the answer using positive exponents

a. $\frac{-3x^{-4}y^5}{-15x^{-5}y^7}$

b. $\frac{24x^6y^{-5}}{8x^{-2}y^2}$

$\frac{-3}{-15} x^{-4-(-5)} y^{5-7}$

$\frac{1}{5} x y^{-2} = \frac{x}{5y^2}$

The Power Rule

For any real number a and any integers m and n for which a^m and $(a^m)^n$:

$(a^m)^n = a^{mn}$

↑ exists ↑ exists

Example: Simplify

a. $(y^2)^6$

b. $(2^3)^4$

c. $(x^5)^3$

d. $(x^{-3})^{-7}$

$= y^{12}$

$= 2^{12}$

$= x^{15}$

$= x^{21}$

Example: Simplify

a. $(3^3)^6$

b. $(x^6)^7$

c. $(x^{-4})^{-5}$

d. $(x^{-6})^4$

Product

Raising a ~~Power~~ or a Quotient to a Power

$$(ab)^m = a^m b^m$$

Raising a product to a power

For any real number a and any integers a and b ~~for which a^m and $(a^m)^n$~~:

a. $(-2x)^3$

$= (-2)^3 x^3$

$= -8x^3$

b. $(-3x^5y^{-1})^{-4}$

$= (-3)^{-4}(x^5)^{-4}(y^{-1})^{-4}$

$= \frac{1}{(-3)^4} \cdot x^{-20} y^4 = \frac{1}{81} x^{-20} y^4$

$= \frac{y^4}{81x^{20}}$

Raising a Quotient to a power

For any integer n and any real numbers a and b for which a/b, a^n, b^n exist:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

a. $\left(\frac{x^2}{2}\right)^4$

$= \frac{(x^2)^4}{2^4}$

$= \frac{x^8}{16}$

b. $\left(\frac{y^2z^3}{5}\right)^{-3}$

$= \frac{(y^2z^3)^{-3}}{5^{-3}}$

$= \frac{(y^2)^{-3}(z^3)^{-3}}{5^{-3}}$

$= \frac{y^{-6}z^{-9}}{5^{-3}}$

$= \frac{5^3}{y^6 z^9}$

$= \frac{125}{y^6 z^9}$