2.2 Functions

Domain and Range

A **Function** is a special type of relationship between two sets. For instance:

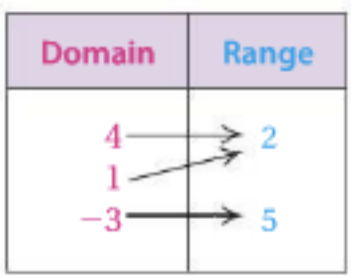
- Each person in a class is matched with one date of birth.
- Each bar code in a store is matched with one price.
- Each real number is matched with its cube.

In each case, the **first set** is called the **domain**, and the **second set** is called the **range**. For every element in the domain, there is **exactly one** corresponding element in the range.

This unique type of correspondence is what we call a **function**.

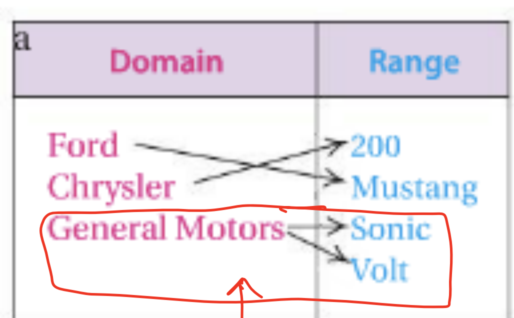
Example 1: Determine whether each correspondence is a function

a)



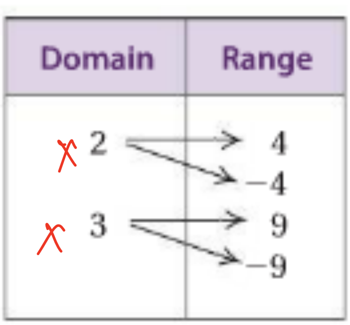
FUNCTION

b)



NOT A FUNCTION

Example 2: Determine whether each correspondence is a function



Not a function

Function

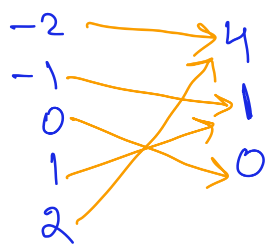
A function is a relationship between two sets: the **domain** (the first set) and the **range** (the second set). Each element of the domain is paired with **exactly one** element of the range.

Example 3: Determine whether each correspondence is a function.

a) The correspondence that assigns to a person his or her weight

Yes

b) The correspondence that assigns to the numbers -2, -1, 0, 1, 2 each numbers square



Yes

c) The correspondence that assigns to a best- selling author the titles of the books written by that author

may have more than one book written by them.

No

Example 4: For the correspondence $\{(-6,7),(1,4),(-3,4),(4,-5)\}$

a) Write the domain

$\{-6, 1, -3, 4\}$

b) Write the range

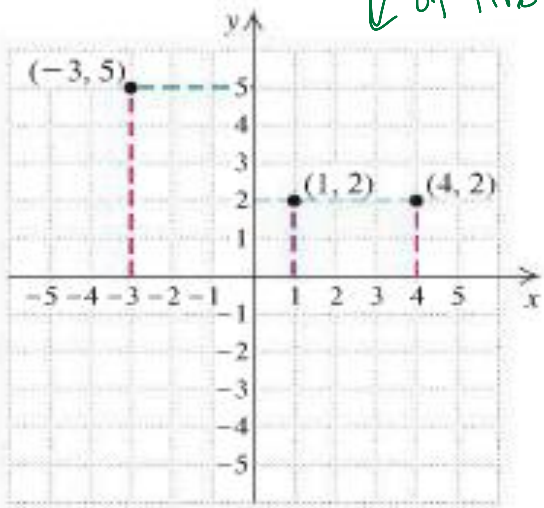
$\{7, 4, -5\}$

Functions and Graphs

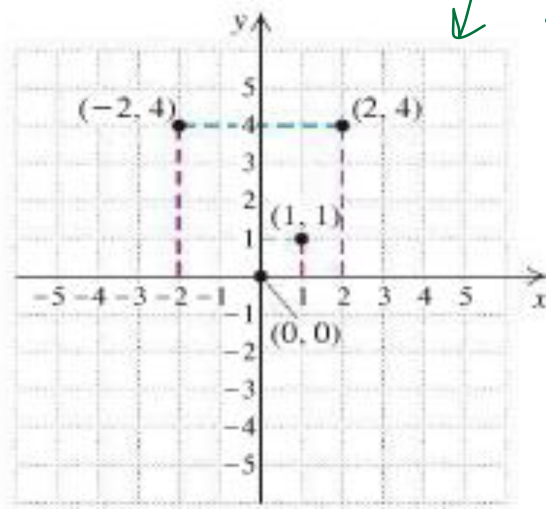
The function in Example 1(a) can be written $\{(-3, 5), (1, 2), (4, 2)\}$ and the function in Example 2(b) $\{(-2, 4), (0, 0), (1, 1), (2, 4)\}$. We graph these functions in black as follows.

graph of first fn.

graph of second fn.



The function $\{(-3, 5), (1, 2), (4, 2)\}$
Domain is $\{-3, 1, 4\}$
Range is $\{5, 2\}$



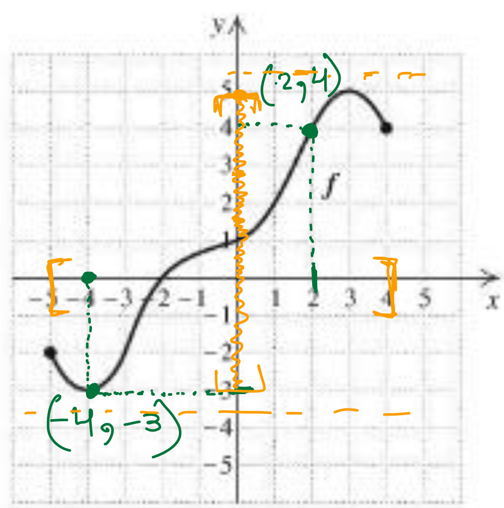
The function $\{(-2, 4), (0, 0), (1, 1), (2, 4)\}$
Domain is $\{-2, 0, 1, 2\}$
Range is $\{4, 0, 1\}$

Example 5: For the function f, determine each of the following.

a) the member of the range that is paired with 2 $x=2$

y-value corresponding to

$=4$



b) the domain of f

every real number greater than or equal to -5 and small than or equal to 4

$\{x : -5 \le x \le 4\}$

c) the member of the domain paired with -3

x-value

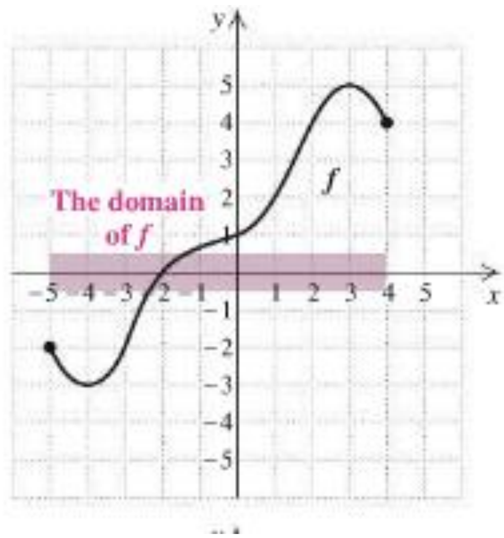
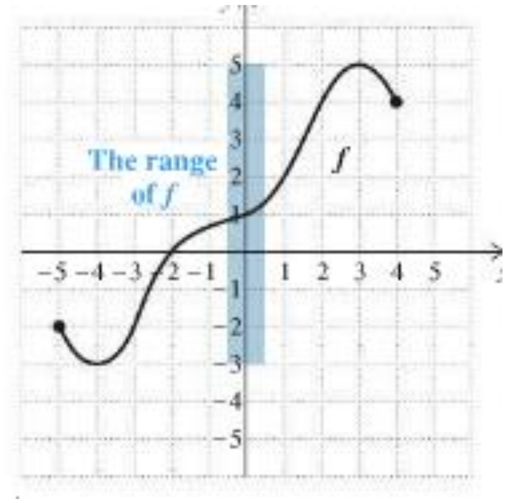
$y=-3$

$= -4$

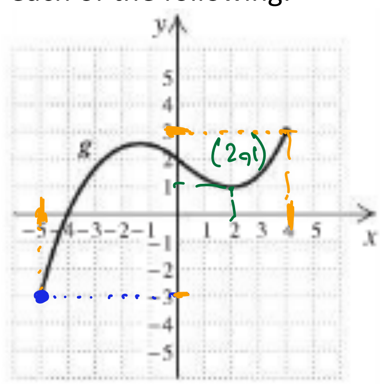
d) the range of f

every real no. greater than or equal to -3 and smaller than or equal to 5

$\{y : -3 \le y \le 5\}$

Domain	Range
	

Example 6: For the function f, determine each of the following.



a) the member of the range that is paired with 2

y-value

$x = 2$

$y = 1$

b) the domain of f

$\{x : -5 \le x \le 4$

c) the member of the domain paired with -3

x-value

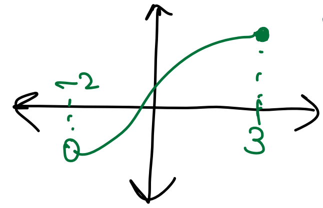
$y = -3$

$x = -5$

d) the range of f

$\{y : -3 \le y \le 3\}$

Note:

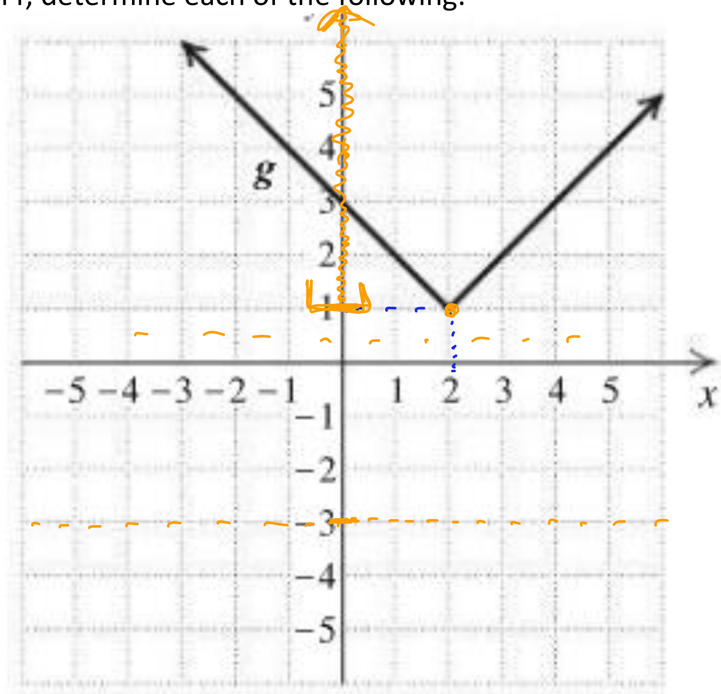


Domain $= \{x : -2 < x \leq 3\}$

A **closed dot** on a graph (as shown in Example 4) means the point **is included** in the function. An **open dot** means the point **is not included** in the function.

In Example 4, the dots also mark the **endpoints** of the graph. A function's domain or range may also extend indefinitely in the positive or negative direction, approaching infinity.

Example 7: For the function f, determine each of the following.



a) the member of the range that is paired with 2

y-value $\qquad x = 2$

$y = 1$

b) the domain of f

Every real number

c) the member of the domain paired with -3

x-value $\qquad y = -3$

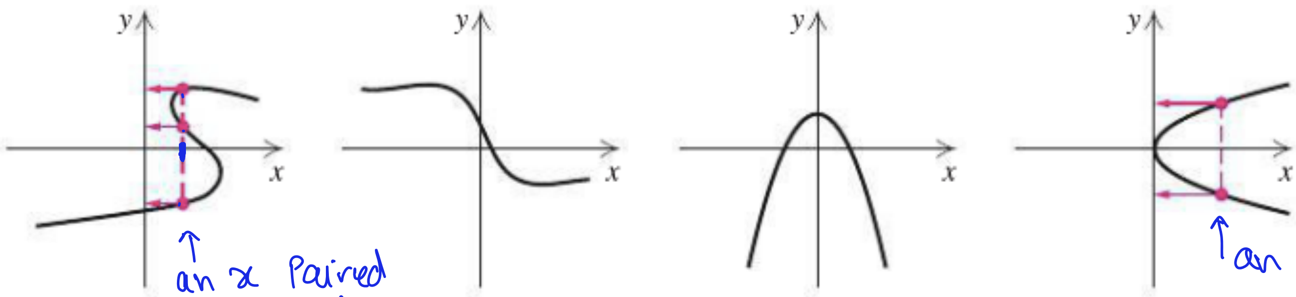
No such member of the domain exists.

d) the range of f

$\{y : y \geq 1\}$

If a vertical line crosses a graph more than once, then the graph does **not** represent a function.

(Vertical Line Test)



an x Paired with 3 y-values

Not a function. Three y-values correspond to one x-value.

A function

A function

an x is Paired with 2 y-values

Not a function. Two y-values correspond to one x-value.

Relation

A relation is a connection between two sets: the **domain** (the first set) and the **range** (the second set). Each element of the domain is paired with **at least one** element of the range.

Function Notation and Equations

- We often think of an element of the domain of a function as an **input** and its corresponding element of the range as an **output**

$$f = \{(-3,1),(1,-2),(3,0),(4,5)\}$$

$f(-3) = 1$ $\quad$ $f(1) = -2$ $\quad$ $f(3) = 0$ $\quad$ $f(4) = 5$

$$\xrightarrow{-3} \boxed{f} \rightarrow 1$$

> ***CAUTION!*** $f(x)$ *does not mean* f times x.

- Most functions are described by equations.

Input

$$f(x) = 2x + 3 \qquad f(4) = 2 \cdot 4 + 3$$

$$= 11. \longleftarrow \text{Output}$$

Double $\quad$ Add 3

$y = 2x + 3$ or $y = f(x)$

Example 7: Find each indicated function value

a) $f(5), for\ f(x) = 3x + 2$

$$f(5) = 3(5) + 2 = 17$$

b) $h(4),\ for\ h(x) = 7$

$$h(4) = 7$$

c) $g(-2), for\ g(r) = 5r^2 + 3r$

$$g(-2) = 5(-2)^2 + 3(-2)$$
$$= 20 - 6 = 14$$

d) $F(a) + 1\ ,\ for\ F(x) = 3x + 2$

$$F(a) = 3a + 2$$
$$F(a) + 1 = 3a + 2 + 1 = 3a + 3$$

$$F(a) + 1 \neq F(a+1)$$

e) $F(a + 1), for\ F(x) = 3x + 2$

$$F(a+1) = 3(a+1) + 2 = 3a + 3 + 2 = 3a + 5$$

f) $F(x + h)$, for $f(x) = 2x - 4$

$$F(x+h) = 2(x+h) - 4 = 2x + 2h - 4$$

g) $F(x + h) - F(x)$, for $f(x) = -x + 5$

$$F(x+h) = -(x+h) + 5 = -x - h + 5$$
$$F(x) = -x + 5$$
$$F(x+h) - F(x) = -x - h + 5 - (-x + 5) = \cancel{-x} - h \cancel{+5} \cancel{+x} \cancel{-5}$$

$= -h$

Example 8: Let $f(x) = 3x - 7$

a) what output corresponds to an input of 5?

y when $x=5$ $x=5$

or $f(5) = 3(5) - 7 = 15 - 7 = 8$

b) what input corresponds to an output of 5?

$x = ?$ $y=5$

Find x for which $f(x) = 5 \Rightarrow 3x - 7 = 5$

$\Rightarrow 3x = 7 + 5 \Rightarrow 3x = 12$

$\Rightarrow x = \frac{12}{3} = 4$

Example 9: For the equation, determine the domain of f

a) $f(x) = |x|$

can be applied to any real number

Domain of f = All real numbers.

b) $f(x) = \frac{x}{2x-6}$

want to make sure that the denominator is not zero.

$2x - 6 = 0 \Rightarrow 2x = 6 \Rightarrow x = 3$

$\Rightarrow$ As long as $x \neq 3$, denominate will not be zero

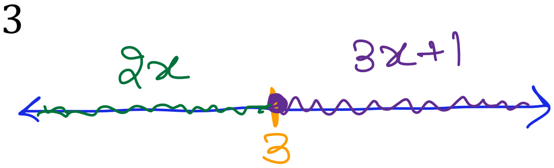
CAUTION! The denominator cannot be 0, but the numerator can be any number.

Domain of f = All real numbers except 3

Piecewise Defined Function

Example 10: Find each function value for the function given

$$f(x) = \begin{cases} 2x\,, if\ x < 3 \\ 3x+1, if\ x \geq 3 \end{cases}$$



a) $f(4)$ $4 > 3 \Rightarrow$ second defn.

$\Rightarrow f(4) = 3(4)+1 = 13$

b) $f(-10)$ $-10 < 3 \Rightarrow$ first defn.

$\Rightarrow f(-10) = 2(-10) = -20$

c) $f(6)$ $6 > 3 \Rightarrow f(6) = 3(6)+1 = 19$

d) $f(3)$ $3 = 3 \Rightarrow f(3) = 3(3)+1 = 10$

e) $f(2)$ $2 < 3 \Rightarrow f(2) = 2(2) = 4$

f) $f(0)$

$0 < 3 \Rightarrow f(0) = 2(0) = 0$