Math -I 110 3.2 Notes

Solving Systems of Equations by Substitution

1. Solve one equation for a variable
2. Substitute expression into the other equation
3. Solve equation
4. Use the variable you know to find the other variable.

Solve the systems by substitution

$\begin{cases} x = 3y \\ 4x + 2y = 70 \end{cases}$

Step 1 $x = 3y$ ✓

Step 2 $4x + 2y = 70$

$\Rightarrow 4(3y) + 2y = 70$ ✓

Step 3 $12y + 2y = 70$

$\Rightarrow 14y = 70 \Rightarrow y = \frac{70}{14} = 5$ ✓

Step 4 $x = 3(5) \Rightarrow x = 15$ ✓

$(x, y) = (15, 5)$

$\begin{cases} y = -5 \\ 7x - y = 12 \end{cases}$

$(1, -5)$

$\begin{cases} x = -4y + 6 \\ 6x - 5y = 7 \end{cases}$

HW

$(x, y) = (2, 1)$

$\begin{cases} y = -2x - 3 \\ -4x - 6y = -6 \end{cases}$

Step 1 $y = -2x - 3$

Step 2 $-4x - 6(-2x - 3) = -6$

Step 3 $-4x + 12x + 18 = -6$

$\Rightarrow 8x = -6 - 18 = -24$

$\Rightarrow x = \frac{-24}{8} = -3$

Step 4 $y = -2(-3) - 3 = 6 - 3 = 3$

$(x, y) = (-3, 3)$

$$\begin{cases} -18x + 3y = 7 \\ y = 6x - 4 \end{cases}$$

$y = 6x - 4$

$\Rightarrow -18x + 3(6x - 4) = 7$

$\Rightarrow -18x + 18x - 12 = 7$

$\Rightarrow -12 = 7$ Contradiction

NO solution

*so these lines are inconsistent

$$\begin{cases} x = 3y + 16 \\ 2x - 6y = 32 \end{cases}$$

$x = 3y + 16$

$\Rightarrow 2(3y + 16) - 6y = 32$

$\Rightarrow 6y + 32 - 6y = 32$

$\Rightarrow 32 = 32$

identity

*so these lines are Consistent dependent | infinite solutions

What is both equations have a variable isolated?

$$\begin{cases} y = -2x \\ y = -8x - 36 \end{cases}$$

$y = -2x$ $\qquad$ $y = -8x - 36$

$-2x = -8x - 36$ $\longleftrightarrow$ same $\longleftrightarrow$ $-8x - 36 = -2x$

$x = -6 \Rightarrow y = -2(-6) = 12$

$\Rightarrow (x, y) = (-6, 12)$

The next two systems address a common mistake.

$$\begin{cases} -7x - y = -24 \\ y = 3x - 16 \end{cases}$$

$y = 3x - 16$

$-7x - (3x - 16) = -24$

$-7x - 3x + 16 = -24$

$-10x = -24 - 16$

$-10x = -40 \Rightarrow x = 4$

$\Rightarrow y = 3(4) - 16 = -4 \Rightarrow (4, -4)$

$$\begin{cases} y = 7x - 21 \\ -3x - y = -19 \end{cases}$$

HW.

$(4, 7)$

The systems below do not have an isolated variable. So decide which variable will be the easiest to isolate

$$\begin{cases} 3x+7y=14 \\ x-5y=12 \end{cases}$$

Step1 $x-5y=12$

$\Rightarrow x=5y+12$

Step2 $3(5y+12)+7y=14$

Step3 $15y+36+7y=14$

$\Rightarrow 22y=14-36=-22$

$\Rightarrow y=\frac{-22}{22}=-1$

Step4 $\Rightarrow x=5(-1)+12=-5+12=7$

$\Rightarrow (7,-1)$

$$\begin{cases} -4x+y=1 \\ -3x+5y=5 \end{cases}$$

Step1 $-4x+y=1 \Rightarrow y=4x+1$

Step2 $-3x+5(4x+1)=5$

Step3 $\Rightarrow -3x+20x+5=5$

$\Rightarrow 17x=5-5=0$

$\Rightarrow x=0$

Step4 $\Rightarrow y=4(0)+1=1$

$(0,1)$

Solving Systems of Equations by Elimination

1. **Create opposite like terms**
2. **Add the equations together thus eliminating one of the variable tems**
3. **Solve the one step equation**
4. **Solve for the other variable**

$$\begin{cases} -2x+y=7 \\ 2x+y=1 \end{cases}$$

add $2y=8 \Rightarrow y=4$

$$\begin{cases} -2x+y=14 \\ 2x+8y=22 \end{cases}$$

add $y+8y=14+22$

$9y=36 \Rightarrow y=4$

$-2x+4=14 \Rightarrow -2x=10$

$\Rightarrow x=-5$

$(-5,4)$

$$\begin{cases} -x+2y=-13 \\ -x-2y=11 \end{cases}$$

HW.

$(1,-6)$

$$\begin{cases} 6x-6y=24 \\ 6x-7y=18 \end{cases}$$

$\times -1$

$-1\times(6x-6y)=24\times-1 \Rightarrow -6x+6y=-24$

add $6x-7y=18$

$-y=-6$

$\Rightarrow y=6$

$\Rightarrow 6x-6(6)=24 \Rightarrow 6x=60 \Rightarrow x=10$

$(10,6)$

$$\begin{cases} 2x-8y=8 \\ -3x-8y=8 \end{cases}$$

$-1(2x-8y)=-1\cdot 8$

$\Rightarrow -2x+8y=-8$

$-3x-8y=8$

$-5x=-8+8=0 \Rightarrow x=0$

$2(0)-8y=8 \Rightarrow -8y=8$

$\Rightarrow y=-1$

$(0,-1)$

$\begin{cases} 4x - 3y = 6 \\ -4x + 3y = -6 \end{cases}$

$0 = 0$

identity

infinite solution

*these lines are Consistent dependent

$\begin{cases} 6x + 7y = 18 \\ 6x - 7y = 23 \end{cases}$

$0 = 18 + 23$

$0 = 41$

Contradiction

no solution

*these lines are Inconsistent

$\begin{cases} -3x + 8y = -13 \\ 12x + 6y = -24 \end{cases}$

$-3x + 8y = -13$ (×4) (×4)

$\Rightarrow -12x + 32y = -52$

$12x + 6y = -24$

$32y + 6y = -52 - 24$

$\Rightarrow 38y = -76$

$\Rightarrow y = \frac{-76}{38} = -2$

first eqn

$-3x + 8(-2) = -13$

$\Rightarrow -3x - 16 = -13 \Rightarrow -3x = 16 - 13$

$\begin{cases} 5x - 7y = 5 \\ 8x - 14y = -6 \end{cases}$ ×-2

$-10x + 14y = -10$

$8x - 14y = -6$

$-10x + 8x = -10 - 6$

$\Rightarrow -2x = -16$

$\Rightarrow x = \frac{-16}{-2} \Rightarrow x = 8$

$\Rightarrow 5x - 7y = 5$

$\Rightarrow 5(8) - 7y = 5 \Rightarrow -7y = 5 - 40$

$\Rightarrow -7y = -35 \Rightarrow y = 5$

$\Rightarrow -3x = 3 \Rightarrow x = -1$

$(-1, -2)$

$(8, 5)$

$\begin{cases} 2x - 3y = -6 \\ -3x + 4y = 11 \end{cases}$

$(2x - 3y = -6) \times 3$ (-ve of the coeff. of x in 2nd eqn)

$\Rightarrow 6x - 9y = -18$

$(-3x + 4y = 11) \times 2$ (coeff. of x in 1st eqn)

$\Rightarrow -6x + 8y = 22$

$6x - 9y = -18$

$-y = 4 \Rightarrow y = -4$

$\begin{cases} 4x - 9y = 5 \rightarrow -3 \\ 3x - 2y = -1 \rightarrow 4 \end{cases}$

$-12x + 27y = -15$

$-12x - 8y = -4$

$\Rightarrow 19y = -19$

$\Rightarrow y = -1$

$3x - 2(-1) = -1$

$3x + 2 = -1$

$3x = -3 \Rightarrow x = -1$

$2x - 3(-4) = -6$

$2x + 12 = -6$

$\Rightarrow 2x = -18 \Rightarrow x = -9$

$(-9, -4)$

In-Class Quiz – 3

(1) [5 Pts] Find whether the given lines are Parallel, Perpendicular or neither.

$2x + y = 1 \longrightarrow y = -2x + 1 \Rightarrow m_1 = -2$

$2x - y = 4 \longrightarrow -y = -2x + 4 \quad m_1 m_2 = -1$

$y = 2x - 4 \Rightarrow m_2 = 2$

(2) [8 Pts] Find the equation of the line perpendicular to the line $2x + y = 1$ and passing through the origin $(0,0)$.

$m_1 = -2$

$m_1 m_2 = -1 \Rightarrow -2 m_2 = -1 \Rightarrow m_2 = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x$

(3) [7 Pts] Let $f(x) = x^2 - 2x + 2$, $g(x) = x^2 + 1$. Find $f(1) + g(1)$ and $g(x) - f(x)$.

$x^2 + 1 - (x^2 - 2x + 2)$

$x^2 + 1 - x^2 + 2x - 2 = 2x - 1$