$3.1, 3.2, 3.3, 3.9, 3.6$
$4.1, 4.2, 4.4$

Math-I 110 3.9 Notes

Break Even Analysis: Profit = Revenue – Cost or $P(x) = R(x) - C(x)$

1. Suppose that for a certain company, $C(x) = 30x + 216{,}000$ represents the total-cost function, and $R(x) = 70x$ represents the total-revenue function.
   a. Find the total-profit function.

   $P(x) = R(x) - C(x) = 70x - (30x + 216000)$
   $= 70x - 30x - 216000$
   $P(x) = 40x - 216000$

   b. Find the break-even point.

   $P(x) = 0$

   $40x - 216000 = 0$

   $\Rightarrow 40x = 216000 \Rightarrow x = \frac{216000}{40} = 5400$

Find the equilibrium point for a pair of supply and demand functions. $D(p) = S(p)$

1. Find the equilibrium point for the following pair of demand and supply functions.

$$D(p) = 2000 - 15p$$
$$S(p) = 680 + 7p$$

$D(P) = S(P)$

$\Rightarrow 2000 - 15P = 680 + 7P$
$-2000 \quad -7P \quad -2000 \quad -7P$

$\Rightarrow -22P = 680 - 2000$

$\Rightarrow -22P = -1320$

$\Rightarrow P = \frac{-1320}{-22} \Rightarrow P = 60$

Application of Business and Economics

1. An electronics company plans to introduce a new laptop computer. The fixed costs are $166,950, and the variable costs are $150 per unit. The revenue from each computer is $600.
   a. Find the total cost function

   $$C(x) = 150x + 166950$$

   b. Find the total revenue function

   $$R(x) = 600x$$

   c. Find the total profit function

   $$P(x) = R(x) - C(x)$$
   $$= 600x - (150x + 166950) = 600x - 150x - 166950$$
   $$= \boxed{450x - 166950}$$

   d. Find the profit or loss of selling 100 computers

   $$P(100)$$

   $$P(100) = 450(100) - 166950 = 45000 - 166950$$
   $$= -121950$$

   loss

   e. Find the profit or loss of selling 500 computers

   $$P(500) = 450(500) - 166950 = 225000 - 166950$$
   $$= 58050$$

   f. Find the break-even point

   $$P(x) = 0$$
   $$450x - 166950 = 0$$
   $$\Rightarrow x = \frac{166950}{450} \Rightarrow \boxed{x = 371}$$

Supply function

2. A company is willing to produce 100 yo-yos at $8.00 each and 500 yo-yos at $14.00 each. Research indicates that the public will buy 500 yo-yos at $7.00 each and 100 yo-yos at $15.00 each. Find the equilibrium point.

demand function

$S(P)$ is a linear function passing through $(8, 100)$, $(14, 500)$

$$m_S = \frac{500-100}{14-8} = \frac{400}{6} = \frac{200}{3} \Rightarrow S - 100 = \frac{200}{3}(P-8) \Rightarrow \boxed{S(P) = \frac{200}{3}(P-8) + 100}$$

$D(P)$ is a linear function passing through $(7, 500)$, $(15, 100)$

$$m_D = \frac{100-500}{15-7} = -\frac{400}{8} = -50 \Rightarrow D - 100 = -50(P-15)$$

$$\Rightarrow \boxed{D(P) = -50(P-15) + 100}$$

Eqm pt $\Rightarrow D(P) = S(P) \Rightarrow -50(P-15) + 100 = \frac{200}{3}(P-8) + 100$

3. ~~Great Foods will soon begin producing a new line of puppy food. The marketing department predicts that the demand function will be D(p) = minus15.32p + 978.74 and the supply function will be S(p) = 93.95pminus 6.37. Answer the following questions.~~

$$-50(P-15) + \cancel{100} = \frac{200}{3}(P-8) + \cancel{100} \Rightarrow -50(P-15) = \frac{200}{3}(P-8)$$

-100 $\quad -100$

$$\Rightarrow P - 15 = \frac{1}{-\cancel{50}} \cdot \frac{\overset{4}{\cancel{200}}}{3}(P-8) \Rightarrow P - 15 = -\frac{4}{3}(P-8)$$

$$\Rightarrow P - 15 = -\frac{4P}{3} + \frac{32}{3} \Rightarrow P + \frac{4P}{3} = \frac{32}{3} + 15 \Rightarrow \frac{7P}{3} = \frac{77}{3}$$

$$\Rightarrow P = \frac{3}{7} \cdot \frac{77}{3} \Rightarrow \boxed{P = 11}$$

4. A 1.5-gallon-per-minute aerator can save up to $125 per year. A shower head, containing this aerator, was bought for $7.12. Assuming savings of $125 per year, how long will it take to break even on the purchase?

Total Cost = Total savings.

Let us suppose we break-even after x years.

$$\Rightarrow 7.12 = 125x$$

$$\Rightarrow x = \frac{7.12}{125} \text{ years.}$$

$$\Rightarrow x = \frac{7.12}{125} \times 365 \text{ days.} \Rightarrow x = 20.79 \simeq 21 \text{ days.}$$