Math-I 110 4.1 Inequalities

Objective: Determine whether a given number is a solution of an inequality.

Example: Determine whether the given values are solutions to the inequality $x - 8 > 2x - 2$

$> \geq < \leq$ } four kinds of inequalities.

a) $x = 3$

$3 - 8 > 2(3) - 2$

$-5 > 4$

$\Rightarrow$ Not a solution.

b) $x = -19$

$-19 - 8 > 2(-19) - 2$

$-27 > -38 - 2$

$-27 > -40$

$\Rightarrow$ Yes, it is a soln.

c) $x = -12$

$-12 - 8 > 2(-12) - 2$

$-20 > -24 - 2$

$-20 > -26$

Yes, it is a soln.

d) $x = -1$

$-1 - 8 > 2(-1) - 2$

$-9 > -4$

Not a soln.

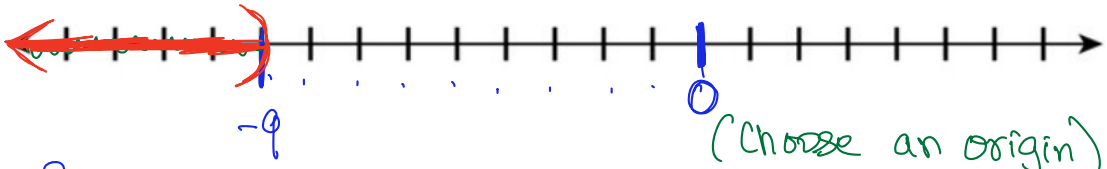
Objective: Write interval notation for and graph solution set to the inequality

Interval Notation	Set-Builder Notation	Graph*
(a, b) open interval	$\{x \mid a < x < b\}$	
$[a, b]$ closed interval	$\{x \mid a \le x \le b\}$	
$(a, b]$ half-open interval	$\{x \mid a < x \le b\}$	
$[a, b)$ half-open interval	$\{x \mid a \le x < b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \ge a\}$	
$(-\infty, a)$	$\{x \mid x < a\}$	
$(-\infty, a]$	$\{x \mid x \le a\}$	

$(-\infty, \infty)$ {All real numbers} 

Example: Graph the inequality and write the solution set using both set-builder notation and interval notation

$$t < -9$$



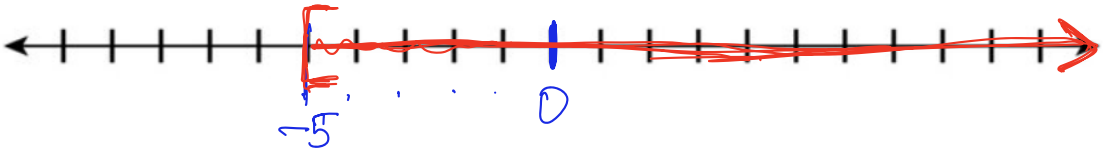
Set builder:

$\{t \mid t < -9\}$

Interval Notation: → $(-\infty, -9)$

Example: Graph the inequality and write the solution set using both set-builder notation and interval notation

$$x \ge -5$$



Set builder:

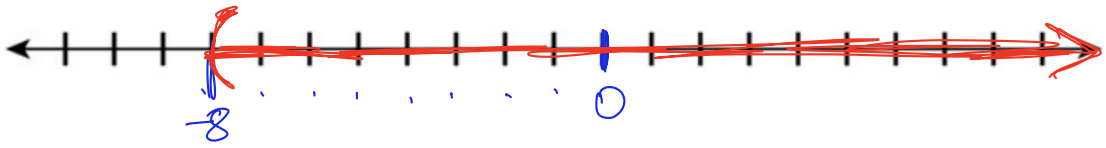
$\{x \mid x \ge -5\}$

Interval Notation:

$[-5, \infty)$

Example: Graph the inequality and write the solution set using both set-builder notation and interval notation

$$y > -8$$



Set builder:

$\{y \mid y > -8\}$

Interval Notation: $(-8, \infty)$

Objective: Solve an inequality using addition principle for inequalities.

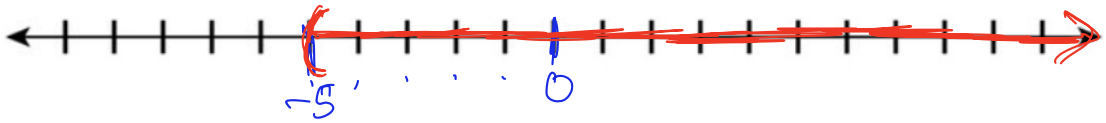
Example: Solve, then graph the inequality and write the solution set using both set-builder notation and interval notation

$x + 8 > 3$

$-8 \quad -8$

$\Rightarrow x > 3 - 8$

$\Rightarrow x > -5$



Set builder:

$\{x \mid x > -5\}$

Interval Notation: $(-5, \infty)$

Example Graph the inequality and write the solution set using both set-builder notation and interval notation

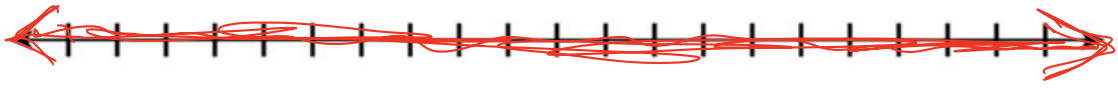
$x + 4 \leq 4 + x$

$-4 \quad -4$

$\Rightarrow x \leq x$

$-x \quad -x$

$\Rightarrow 0 \leq 0$ identity



Set builder:

$\{x \mid x$ is any real number$\}$

Interval Notation:

$(-\infty, \infty)$

multiplication with 0 not allowed

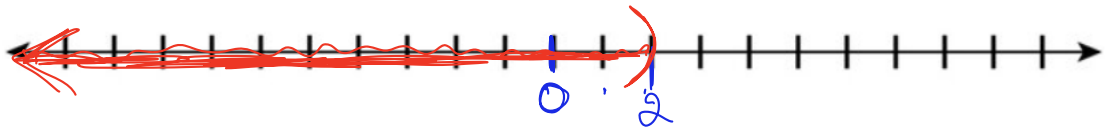
$x = x+1$

$0 \cdot x = 0 \cdot (x+1) \Rightarrow 0 = 0$

Objective: Solve an inequality using multiplication principle for inequalities.

Example Solve, then graph the inequality and write the solution set using both set-builder notation and interval notation

$$\frac{3}{4}y < \frac{3}{2} \Rightarrow \frac{4}{3}\cdot\frac{3}{4}y < \frac{4}{2}\cdot\frac{3}{2} \Rightarrow y < 2$$



Set builder:

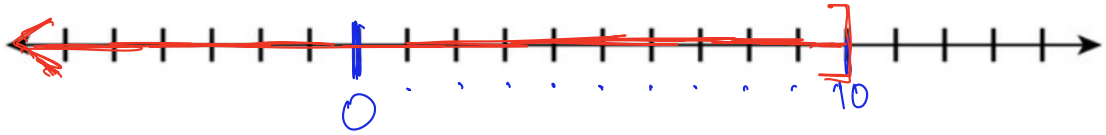
$\{y \mid y < 2\}$

Interval Notation: $(-\infty, 2)$

Example Graph the inequality and write the solution set using both set-builder notation and interval notation

$$-\frac{1}{5}x \geq -2 \Rightarrow -5\cdot\frac{-1}{5}\cdot x \leq -5\cdot(-2) \Rightarrow x \leq 10$$

(-ve) (-ve)



Set builder:

$\{x \mid x \leq 10\}$

Interval Notation: $(-\infty, 10]$

Example Graph the inequality and write the solution set using both set-builder notation and interval notation

$$-5x \geq -80 \qquad \frac{-1}{5}\cdot -5x \leq \frac{-1}{5}\cdot(-80) \Rightarrow x \leq 16$$

Set builder:

$\{x \mid x \leq 16\}$

Interval Notation:

$(-\infty, 16]$

Objective: Using the principles together.

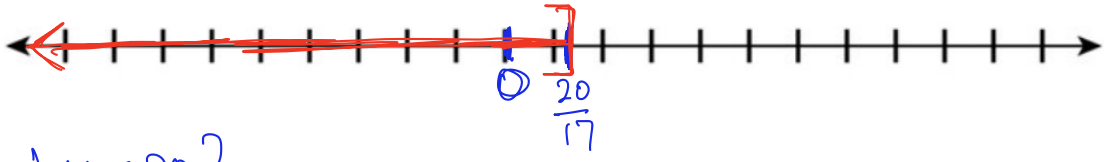
Example Solve, then graph the inequality and write the solution set using both set-builder notation and interval notation

$16 - 7y \geq 10y - 4$ $\Rightarrow -7y \geq 10y - 4 - 16$

$-16 \quad -16 \qquad -10y \quad -10y$

$\Rightarrow -7y - 10y \geq -20 \Rightarrow -17y \geq -20 \Rightarrow \frac{-1}{17} \cdot -17y \leq \frac{-1}{17} \cdot -20$

$\Rightarrow y \leq \frac{20}{17}$



Set builder: $\{y \mid y \leq \frac{20}{17}\}$

Interval Notation: $\longrightarrow (-\infty, \frac{20}{17}]$

Example Solve, then graph the inequality and write the solution set using both set-builder notation and interval notation

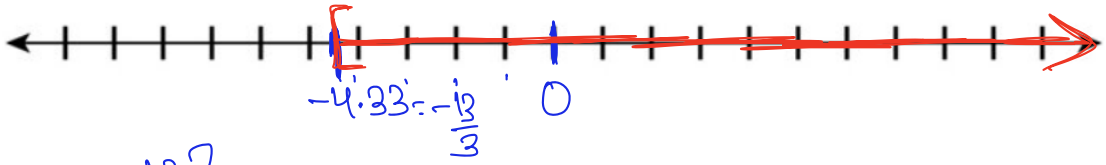
$\frac{3x - 7}{-4} \leq 5$ $\Rightarrow -4 \cdot \frac{3x-7}{-4} \geq -4 \cdot 5$

$\Rightarrow 3x - 7 \geq -20 \Rightarrow 3x \geq -13$

$\Rightarrow x \geq -\frac{13}{3}$

$\downarrow$

-4.33



Set builder: $\{x \mid x \geq -\frac{13}{3}\}$

Interval Notation: $[-\frac{13}{3}, \infty)$

Example: Let $f(x) = -3(x+8) - 5x$ and $g(x) = 4x - 9$. Find all x for which $f(x) > g(x)$

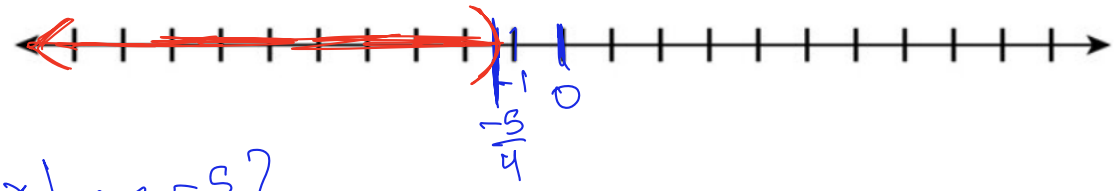
$\Rightarrow -3(x+8) - 5x > 4x - 9$

$\Rightarrow -3x - 24 - 5x > 4x - 9 \Rightarrow -8x > 4x - 9 + 24$

$+24$ $+24$ $-4x$ $-4x$

$\Rightarrow -12x > 15 \Rightarrow \frac{-1}{12} \cdot -12x < \frac{-1}{12} \cdot 15 \Rightarrow x < \frac{-5}{4}$

$= -1.25$



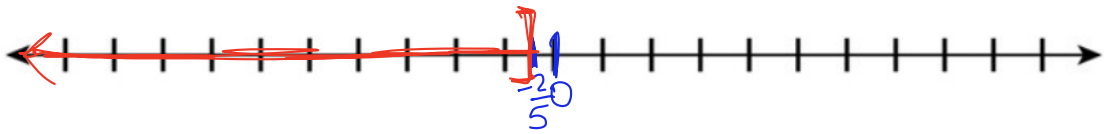
Set builder:

$\{x \mid x < \frac{-5}{4}\}$

Interval Notation:

$(-\infty, \frac{-5}{4})$

Example: Let $f(x) = 8x - 9$ and $g(x) = 3x - 11$. Find all x for which $f(x) \leq g(x)$



Set builder:

$\{x \mid x \leq \frac{-2}{5}\}$

Interval Notation:

$(-\infty, \frac{2}{5}]$

Problem Solving

Write these as an inequality

A number is less than 10. $x < 10$

A number is greater than or equal to 4. $x \geq 4$

The temperature is at most -3 degrees Celsius. $x \leq -3$

A full-time student must take at least 12 credits of classes. $x \geq 12$ x : #credits

The age of the Mayan altar exceeds 1200 years. $x > 1200$

The time of the test was between 45 minutes and 55 minutes. $45 \leq x \leq 55$

Focus-group sessions should last no more than 2 hours. $x \leq 2$

Angenita earns no less than 22 dollars per hour. $x \geq 22$

To rent a car, a driver must have a minimum of 5 years of driving experience. $x \geq 5$

The maximum safe level for chronic inhalation of formaldehyde is 0.003 part per million. $x \leq 0.003$

Data: U.S. Environmental Protection Agency

The costs of production of the software cannot exceed 12,500 dollars. $x \leq 12500$

The number of volunteers was at most 20. $x \leq 20$

Solve

1. **Photography.** Bon-Hwa will photograph a wedding for a flat fee of $900 or for an hourly rate of $120. For what lengths of time would the hourly rate be less expensive?

Let x be # hours for which we need photographer

$120x < 900 \Rightarrow x < \frac{900}{120} \Rightarrow x < 7.5$

(# hours cannot be negative) $(0, 7.5)$

2. **Truck Rentals.** Jenn can rent a moving truck for either $99 with unlimited mileage or $49 plus 80 cents per mile. For what mileages would the unlimited mileage plan save money?

Let x be the mileage needed.

$99 < 49 + 0.8x \Rightarrow -0.8x + 99 < 49$

$-0.8x \quad -0.8x \quad -99 \quad -99$

$\Rightarrow -0.8x < -50 \Rightarrow \frac{-0.8x}{-0.8} > \frac{-50}{-0.8}$

$\Rightarrow x > 62.5 \quad \Rightarrow (62.5, \infty)$

3. **Graduate School.** Unconditional acceptance into the Master of Business Administration (MBA) program at Arkansas Tech University is awarded to those students whose GMAT score plus 200 times their undergraduate grade point average is at least 1000. Chloe's GMAT score was 500. What must her grade point average be in order that she be unconditionally accepted into the program?
Data: atu.edu

Let Chloe's gpa be x

$500 + 200x \geq 1000 \Rightarrow 200x \geq 500$

$\Rightarrow x \geq \frac{500}{200} \Rightarrow x \geq 2.5$

$\Rightarrow [2.5, 4]$

4. **Car Payments**. Some financial advisors suggest that debt payments (other than mortgages) should be less than 8% of a consumer's monthly gross income. Oliver makes $54,000 per year and has a $100 student-loan payment every month. What size car payment can he afford?
Data: thenest.com

$\$x$ per month

$\underbrace{x+100}_{\text{net debth per month}} < \frac{8}{100} \cdot \frac{54000}{12} \Rightarrow x+100 < 360$

$\Rightarrow x < 360 - 100$

$\Rightarrow x < 260$

$\Rightarrow (0, 260)$

5. **Exam Scores**. There are 80 questions on a college entrance examination. Two points are awarded for each correct answer, and one-half point is deducted for each incorrect answer. How many questions does Tami need to answer correctly in order to score at least 100 on the test? Assume that Tami answers every question.

Let x be correct answers

Total of 80 questions. $\Rightarrow$ # incorrect $= 80 - x$

score ≥ 100

$\text{score} = 2x - \frac{1}{2}(80-x) = 2x - 40 + \frac{1}{2}x = \frac{5}{2}x - 40$

$\frac{5}{2}x - 40 \geq 100 \Rightarrow \frac{5}{2}x \geq 140 \Rightarrow x \geq \frac{2}{5} \cdot 140$

$\Rightarrow x \geq 56 \Rightarrow [56, 80]$

6. **Insurance Claims**. After a serious automobile accident, most insurance companies will replace the damaged car with a new one if repair costs exceed 80% of the NADA, or "blue-book," value of the car. Lorenzo's car recently sustained $9200 worth of damage but was not replaced. What was the blue-book value of his car?

x

$9200 \leq \frac{80}{100} x \Rightarrow \frac{80}{100} x \geq 9200$

$\Rightarrow x \geq \frac{100}{80} \cdot 9200 \Rightarrow x \geq 11500$

$\Rightarrow [11500, \infty)$

7. **Well Drilling.** Star Well Drilling offers two plans. Under the "pay-as-you-go" plan, they charge $500 plus $20 per foot for a well of any depth. Under their "guaranteed-water" plan, they charge a flat fee of $9000 for a well that is guaranteed to provide adequate water for a household. For what depths would it save a customer money to use the pay-as-you-go plan?

Let depth be x feets

$\Rightarrow$ Pay-as-you-go plan $= 500 + 20x$

guaranteed-water Plan $= 9000$

$\Rightarrow 20x + 500 < 9000$

hw.

8. **Legal Fees.** Bridgewater Legal Offices charges a $250 retainer fee for real estate transactions plus $180 per hour. Dockside Legal charges a $100 retainer fee plus $230 per hour. For what number of hours does Bridgewater charge more?

[x hours]

Bridgewater $\rightarrow 250 + 180x$

Dockside $\rightarrow 100 + 230x$

$250 + 180x > 100 + 230x$

$-250 \quad -250$

$\Rightarrow 180x > 230x + 100 - 250$

$-230x \quad -230x$

$\Rightarrow -50x > -150$

$\Rightarrow \frac{-50x}{-50} < \frac{-150}{-50}$

$\Rightarrow x < 3$

$\Rightarrow (0, 3)$

9. **Wages.** Toni can be paid in one of two ways:
Plan A: A salary of $2500 per month, plus a commission of 8% of gross sales;
Plan B: A salary of $2800 per month, plus a commission of 5% of gross sales.
For what amount of gross sales should Toni select plan A?

Let x be the gross sales in a month.

Plan A: $2500 + \frac{8}{100} x$, Plan B: $2800 + \frac{5}{100} x$

$\Rightarrow 2500 + \frac{8}{100} x > 2800 + \frac{5}{100} x \Rightarrow \frac{3}{100} x > 300$

$-2500 \quad -\frac{5}{100} x \quad -2500 \quad -\frac{5}{100} x$

$\Rightarrow x > \frac{100}{3} \cdot 300$

$\Rightarrow x > 10000$

$x - 2 < -9$

what can x be?

a) -7

$x = -7 \Rightarrow -7 - 2 = -9$

b) -6

c) -5

> -9

$-8 - 2 < -9 \Rightarrow -10 < -9$

d) -8

Sep 30

In-Class Quiz 5

(1) Let $A = \underset{2\times1}{\begin{bmatrix} 2 \\ 0 \end{bmatrix}}$, $B = \underset{2\times2}{\begin{bmatrix} 1 & -1 \\ 3 & 5 \end{bmatrix}}$,

$C = \underset{1\times2}{\begin{bmatrix} 3 & -3 \end{bmatrix}}$, $D = \underset{2\times2}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$.

5pts (a) $A + C$ — Not Possible

5pts (b) $B + D$

5pts (C) CD

5pts (d) BD

If addition is not possible, you write NOT POSSIBLE

$A \to 2\times1$, $B \to 1\times2$ ← cannot be added

(*) To add/subtract both dimensions should match.

$(m\times n) \pm (m\times n)$

$B = \underset{2\times2}{\begin{bmatrix} 1 & -1 \\ 3 & 5 \end{bmatrix}}$, $D = \underset{2\times2}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$.

(b) $B + D = \begin{bmatrix} 1+0 & -1+1 \\ 3+1 & 5+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11} \\ a_{21} + b_{21} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}_{2\times 1}$$

2×2 $\quad$ $m \times n = 2 \times n$ $\quad$ (m = 2)

$(p \times q) \cdot (m \times n)$ $\quad$ $q = m$

$AB \rightarrow 2 \times 1$

$$AB = \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} \end{bmatrix}$$

Let $A = \underset{2\times 1}{\begin{bmatrix} 2 \\ 0 \end{bmatrix}}$, $B = \underset{2\times 2}{\begin{bmatrix} 1 & -1 \\ 3 & 5 \end{bmatrix}}$,

$C = \underset{1\times 2}{\begin{bmatrix} 3 & -3 \end{bmatrix}}$, $D = \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$.

$AC = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \end{bmatrix}$ (2×2; 2×1 · 1×2)

$= \begin{bmatrix} 2\cdot 3 & 2\cdot(-3) \\ 0\cdot 3 & 0\cdot(-3) \end{bmatrix}$

$= \begin{bmatrix} 6 & -6 \\ 0 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & -1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ (2×1; 2×2 · 2×1)

$= \begin{bmatrix} 1(2)+(-1)(0) \\ 3(2)+5(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

$BD = \begin{bmatrix} 1 & -1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (2×2; 2×2 · 2×2)

$= \begin{bmatrix} 1(0)+(-1)\cdot 1 & 1(1)+(-1)\cdot 0 \\ 3(0)+5(1) & 3(1)+5(0) \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$

$$DB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 0(1)+1(3) & 0(-1)+1(5) \\ 1(1)+0(3) & 1(-1)+0(5) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$