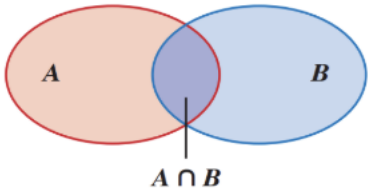
Math-I 110 4.2 Intersections, Unions and Compound Inequalities

and or

Intersection of Sets and Conjunctions of Sentences

The intersection of sets A and B is the set of all elements that are common to both A and B. We denote the intersection of sets A and B as $A \cap B$.



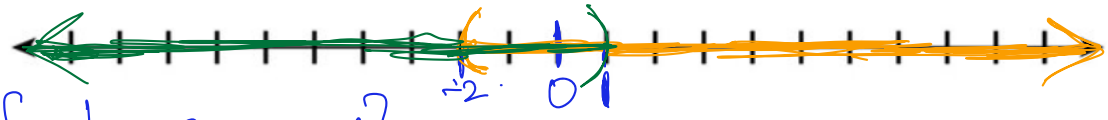
Example: Find the intersection of $\{1,2,3,4,5\} \cap \{-2,-1,0,1,2,3\}$

$= \{1, 2, 3\}$

Example: Graph and write set-builder notation and interval notation for the conjunction

$-2 < x$ and $x < 1$

$(-2, \infty)$ $(-\infty, 1)$



Set builder: $\{x \mid -2 < x < 1\}$

Interval Notation: $(-2, 1)$

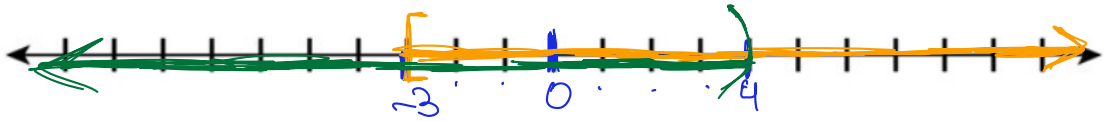
Example: Graph and write set-builder notation and interval notation for the conjunction

$-1 \leq 2x + 5 < 13$

$2x+5 \geq -1$ and $2x+5 < 13 \Rightarrow 2x < 8$

$\Rightarrow 2x \geq -6$ $\Rightarrow x < 4$

$\Rightarrow x \geq -3$



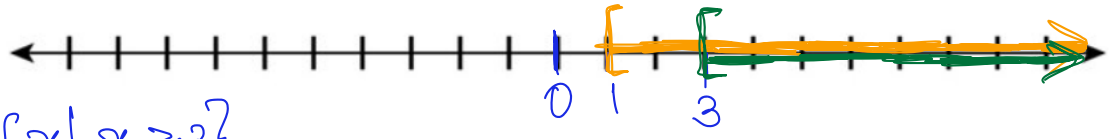
Set builder: $\{x \mid -3 \leq x < 4\}$

Interval Notation: $[-3, 4)$

Example: Solve and graph $2x - 5 \geq -3$ and $5x + 2 \geq 17$. Write the answer using bother ser-builder notation and interval notation.

$2x - 5 \geq -3$ $\qquad$ $5x + 2 \geq 17$

$x \geq 1$ $\qquad$ $x \geq 3$



Set builder: $\{x \mid x \geq 3\}$

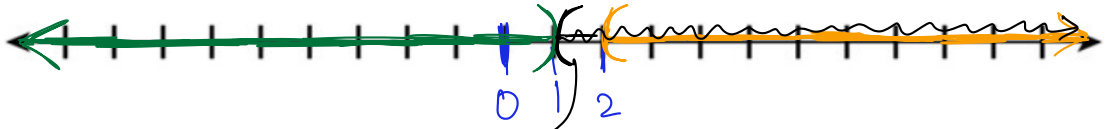
Interval Notation: $[3, \infty)$

Example: Solve and graph $2x - 3 > 1$ and $3x - 1 < 2$. Write the answer using bother ser-builder notation and interval notation.

$2x - 1 > 1$
$\Rightarrow 2x > 2$
$\Rightarrow x > 1$

$2x - 3 > 1$
$\Rightarrow 2x > 4$
$\Rightarrow x > 2$

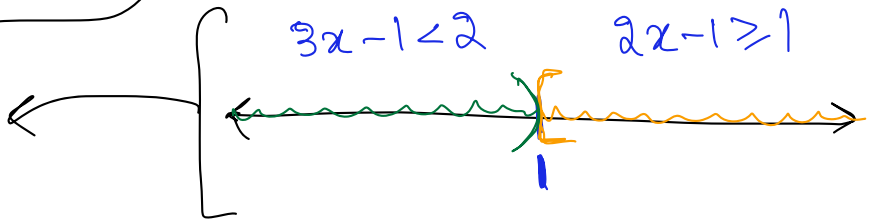
$3x - 1 < 2$
$\Rightarrow 3x < 3$
$\Rightarrow x < 1$

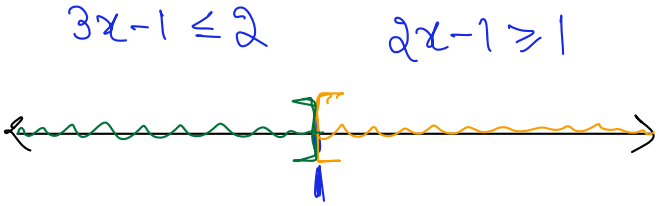


Set builder: Empty set

Interval Notation: No solutions

No such interval.





$\{x \mid x = 1\}$

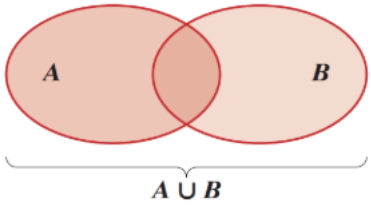
$[1, 1] = \{1\}$

Either A OR B

Unions of sets and disjunctions of Sentences

OR

The union of sets A and B is the collection of elements belonging to A or B. This includes the elements belonging to A and B. We denote the union of A and B by A ∪ B.



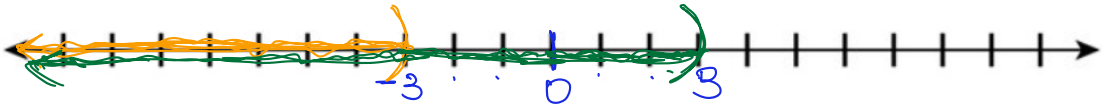
$x \geq 2$

$[x > 2 \text{ or } x = 2]$

Example: Find the union: {2,3,4} ∪ {3,5,7}

$\{2, 3, 4, 5, 7\}$

Example: Graph and write set-builder notation and interval notation for the disjunction $x < -3$ or $x < 3$



Set builder: $\{x \mid x < 3\}$

Interval Notation: $(-\infty, 3)$

Example: Graph and write set-builder notation and interval notation for the disjunction

$7 + 3x < 3$ or $13 - 5x \leq 3$

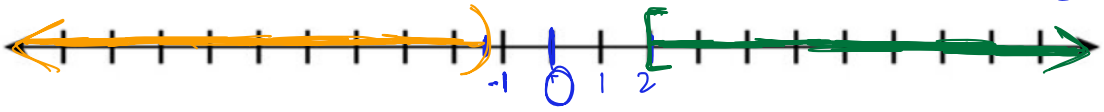
$3x < 3 - 7$

$\Rightarrow x < -\frac{4}{3} = -1.33$

$-5x \leq 3 - 13$

$\Rightarrow -5x \leq -10$

$\Rightarrow \frac{-5x}{-5} \geq \frac{-10}{-5} \Rightarrow x \geq 2$



Set builder: $\{x \mid x < -\frac{4}{3} \text{ or } x \geq 2\}$

Interval Notation:

$(-\infty, -\frac{4}{3}) \cup [2, \infty)$

Example: Graph and write set-builder notation and interval notation for the disjunction

$3x - 11 < 4$ or $4x + 9 \geq 1$

$3x - 11 < 4$

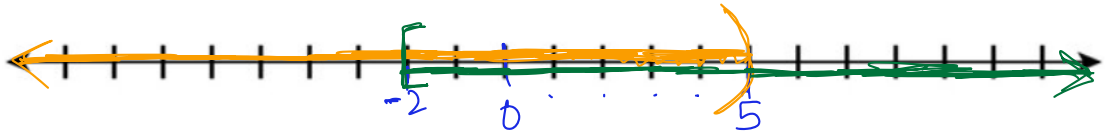
$\Rightarrow 3x < 15$

$\Rightarrow x < 5$

$4x + 9 \geq 1$

$4x \geq -8$

$x \geq -2$



Set builder: $\{x \mid x \text{ is a real number}\}$

Interval Notation: $(-\infty, \infty)$

Interval Notation and Domains

Find the domain of $f(x)$. Use interval notation.

a) $f(x) = \sqrt{7 - x}$

$7 - x \geq 0 \Rightarrow -x \geq -7$

$\Rightarrow \frac{-x}{-1} \leq \frac{-7}{-1} \Rightarrow x \leq 7$

Domain $= (-\infty, 7]$

b) $f(x) = \sqrt{2x + 7}$

$2x + 7 \geq 0 \Rightarrow 2x \geq -7 \Rightarrow x \geq \frac{-7}{2}$

Domain $= \left[\frac{-7}{2}, \infty\right)$