5.1, 5.2, 5.3, 5.4, 5.5

Math-I 110 5.1 Notes

Polynomials → algebraic expression involving only positive powers of the variables and/or constants

1. Terms – the building blocks

- Think of terms as the "words" that make up math expressions.
- A term can be:
- A number (like 7 or 13)
- A variable (like x or y)
- A number and variable multiplied together (like $4w^2$ or $3x$)
- A product of several variables (like x^2y^6)

$5x^5$, 2, $4x$

2. Monomials – multiplication only

- A monomial is a special kind of term. It's made only by multiplying numbers and variables—no variables in the denominator.
- ✅ Examples: x^2y^6, $4w^2$, 13 → monomials
- ❌ Not a monomial: $\boxed{7/x^4}$ (because of the variable in the denominator) → we cannot have negative powers.

3. Degree – the "power level" of a monomial

- The degree of a monomial is the sum of the exponents on its variables.
- $4w^2$ → degree 2
- x^2y^6 → degree 8 (2 + 6 = 8)
- A constant like 13 has degree 0 (nonzero constant)
- The term 0 is a special case: it has no degree

$x^3y^6z^2$

degree = sum of powers of all variables

$= 3 + 6 + 2$

$= 11$

Quick Recap

- Term = a single piece of an expression
- Monomial = a term with multiplication only (no division by variables)
- Degree = add up the exponents to measure its "power"

4. Coefficients

- In the term $4w^2$, the number 4 is called the coefficient.
- A coefficient is the number in front of the variable(s).
- If the term is just a constant (like 7), the coefficient is simply that number.

Coeff. of $7x^5$ is 7

Coeff. of 5 is 5

Coeff. of x^2y^6 is 1

5. Polynomials

- A polynomial is either:
- A single monomial, or
- A sum of monomials
- Examples of polynomials: $\underbrace{3x^2 + 5x - 7}$, $\underbrace{y^3 - 4y}$, $\underbrace{8}$

Special names:

- A polynomial with two terms is a binomial.
- A polynomial with three terms is a trinomial.
- Any polynomial with more than 3 terms is a polynomial with no special name

Degree and Coefficients

The leading term of a polynomial is the term of highest degree. Its coefficient is called the leading coefficient. The degree of a polynomial is the same as the degree of its leading term.

We generally arrange polynomials in one variable so that the exponents *decrease* from left to right. This is called **descending order**. A polynomial with exponents *increasing* from left to right is written in **ascending order**.

Example: Write the polynomial in descending order. Then, identify the leading term, leading coefficient and degree of the polynomial.

$$\underbrace{5x^5}_{5} - \underbrace{7x}_{1} + \underbrace{9x^4}_{4} - \underbrace{6x^3}_{3} + \underbrace{10}_{0}$$

Polynomial in descending order $5x^5 + 9x^4 - 6x^3 - 7x + 10$

Leading term $5x^5$

Leading Coefficient 5

Degree 5

Example: Write the polynomial in descending order. Then, identify the leading term, leading coefficient and degree of the polynomial.

$$\underbrace{5x^4y}_{5} + \underbrace{11x^2y^4}_{6} - \underbrace{2x^3}_{3} + \underbrace{xy}_{2}$$

Polynomial in descending order $11x^2y^4 + 5x^4y - 2x^3 + xy$

Leading term $11x^2y^4$

Leading Coefficient 11

Degree 6

Example: Write the coefficient of each term of the polynomial

$$4x^4 - 6x^3 + 8x$$

First term coefficient 4

Second term coefficient -6

Third term coefficient 8

Example: Determine the degree of each term of the polynomial

$$9x^3y^4 - 5x^4y^6 + 6y^2$$

Degree of first term 7

Degree of second term 10

Degree of third term 2

Classify the Polynomial by the number of terms

1. $9x^4$ 1
2. $2x + 5$ 2
3. $x^2 - 3x + 1$ 3
4. $4x^3 + 2x^2 - x + 6$ 4

Polynomial Functions

A polynomial of degree 0 or 1 is called **linear**. A polynomial in one variable is said to be **quadratic** if it is of degree 2, **cubic** if it is of degree 3, and **quartic** if it is of degree 4.

A *polynomial function* is a function in which ordered pairs are determined by evaluating a polynomial. For example, the function *P* given by $P(x) = 3x^3 - 4x + 6$ is a polynomial function.

Example: Evaluate the polynomial function for $f(-1)$ and $f(0)$

$$f(x) = -x^3 + 8x^2 - 2x + 7$$

$f(-1) = -(-1)^3 + 8(-1)^2 - 2(-1) + 7 = -1 + 8 + 2 + 7 = 16$

$f(0) = -0^3 + 8(0^2) - 2(0) + 7 = 7$

Example: Evaluate the polynomial function for $P(4)$ and $P(0)$

$$P(x) = 4x^2 - 12x + 5$$

$P(4) = 4(4)^2 - 12(4) + 5 = 21$

$P(0) = 4(0)^2 - 12(0) + 5 = 5$

Adding Polynomials

x^2y and xy^2 Not Similar.

terms for which each of variables have same degrees

When two terms have the same variable(s) raised to the same power(s), they are similar, or like, terms and can be "combined."

The sum of two polynomials can be found by writing a plus sign between them and then combining like terms.

Example: Combine like terms

a. $12x + 7 - 7x + 7x^3 - 6x - 6$	b. $12v^2t + 7t^2 - 36v^2t - 9t^2$
$7x^3 + 12x - 7x - 6x + 7 - 6$ $= 7x^3 - x + 1$	$12v^2t - 36v^2t + 7t^2 - 9t^2$ $= -24v^2t - 2t^2$

Example: Add

a. $(-7x^4 + x^2 - 5x) + (-5x^4 + 6x^2 - 9)$	b. $(x^2 + 3x - 7xy - 9) + (-3x^2 - x + 2y^2 + 7)$
$-7x^4 + x^2 - 5x - 5x^4 + 6x^2 - 9$ $= -7x^4 - 5x^4 + x^2 + 6x^2 - 5x - 9$ $= -12x^4 + 7x^2 - 5x - 9$	$x^2 + 3x - 7xy - 9 - 3x^2 - x + 2y^2 + 7$ $x^2 - 3x^2 - 7xy + 2y^2 + 3x - x - 9 + 7$ $= -2x^2 - 7xy + 2y^2 + 2x - 2$

c. $(13s^2d - 3sd^2 + 2sd) + (-10s^2d - 6sd^2 + 8sd)$

$= \underbrace{13s^2d} \underbrace{- 3sd^2} + \underbrace{2sd} \underbrace{- 10s^2d} \underbrace{- 6sd^2} + \underbrace{8sd}$

$= \underbrace{13s^2d - 10s^2d} \underbrace{- 3sd^2 - 6sd^2} \underbrace{+ 2sd + 8sd}$

$= 3s^2d - 9sd^2 + 10sd$

d. $(3a^2 + 13a - 9) + (6a^2 - 3a + 6) + (a^2 - 7a - 7)$

$= 10a^2 + 3a - 10$

Opposites and Subtraction

- If the sum of two polynomials is 0, the polynomials are opposites, or additive inverses, of each other.
- To form the opposite of a polynomial, we can think of distributing the "–" sign, or multiplying each term of the polynomial by –1, and removing the parentheses.
- The opposite of a polynomial P can be written as –P or, equivalently, by replacing each term with its opposite.
- To subtract a polynomial, we add its opposite.

Example: Write two expressions, one with parentheses and one without, for the opposite of the given polynomial. $5x^4 + 2x^2 - 9x - 1$

a. Write the expression with parentheses for the opposite of the polynomial.

$-(5x^4 + 2x^2 - 9x - 1)$

$-(5x^4 + 2x^2 - 9x - 1)$

b. Write the expression without parentheses for the opposite of the polynomial

$-5x^4 - 2x^2 + 9x + 1$

Example: Subtract the polynomials

a. $(7v^2 + 7v + 9) - (9v^2 + 9v - 2)$

$7v^2 + 7v + 9 - 9v^2 - 9v + 2$

$= 7v^2 - 9v^2 + 7v - 9v + 9 + 2$

$= -2v^2 - 2v + 11$

b. $(6a - 5b + c) - (2z + 2b - 7c)$

$6a - 5b + c - 2z - 2b + 7c$

$= 6a - 5b - 2b + c + 7c - 2z$

$= 6a - 7b + 8c - 2z$

c. $(4x^2 + 10xy - 3y^2) - (7x^2 - 8xy + 11y^2)$

$= 4x^2 + 10xy - 3y^2 - 7x^2 + 8xy - 11y^2$

$= -3x^2 + 18xy - 14y^2$

Example: Perform the indicated operations

a. $(7x^2 + 6) - (4x^2 + 1) + (x^2 + 4x)$

$= 7x^2 + 6 - 4x^2 - 1 + x^2 + 4x$

$= 4x^2 + 4x + 5$

b. $(3r^2 - 3r) - (3r - 5) + (7r^2 - 9)$

$= 3r^2 - 3r - 3r + 5 + 7r^2 - 9$

$= 10r^2 - 6r - 4$

c. $(x^2 - 7x + 8) + (5x^2 - 3) - (x^2 - 7x + 8)$

$= \cancel{x^2} - \cancel{7x} + \cancel{8} + 5x^2 - 3 - \cancel{x^2} + \cancel{7x} - \cancel{8}$

$= 5x^2 - 3$

Applications

a. For a rugby club consisting of p people, the number of ways N in which a president, vice president, and treasurer can be elected can be determined using the following function.

$$N(p) = p^3 - 3p + 2p$$] cubic function of p.

The rugby club has 23 members. In how many ways can they elect a president, vice president, and treasurer?

$p = 23$

$N(23) = 23^3 - 3(23) + 2(23) = 12144$

b. The amount of horsepower needed to overcome air resistance by a race car traveling v miles per hour can be approximated by the following polynomial function.

$$h(v) = \frac{0.354}{8250} v^2$$] quadratic

How much horsepower does the race car traveling $\underbrace{\text{190 mph}}_{v = 190}$ need to overcome air resistance?

$h(190) = \frac{0.354}{8250} (190)^2 = 1.55$ horsepower

c. Total profit is defined as total revenue minus total cost. Let the revenue from the sale of x futons be defined as shown below.

$$R(x) = 230\,x - 0.4x^2$$

Let the cost of producing x futons be defined as shown below.

$$C(x) = 7000 + 0.9x^2$$

Find the profit from the sale of 110 futons.

$x = 110$

$$P(x) = R(x) - C(x)$$

$$= (230x - 0.4x^2) - (7000 + 0.9x^2)$$

$$= 230x - 0.4x^2 - 7000 - 0.9x^2$$

$$= -1.3x^2 + 230x - 7000$$

$$P(110) = -1.3(110)^2 + 230(110) - 7000$$

$$= 2570$$

$\boxed{\text{Quiz 6}}$

or → $\cup$ (union) , and → $\cap$ (intersecti-on)

1. Solve the following inequalities :

5pts (a) $3x + 3 > 6$ and $2x \leq 4$

5pts (b) $2(x-1) < 6$ or $4x + 5 > 25$

5pts (c) $2x + 3x \geq 10$ and $x - 2 \leq 0$

5pts (d) $\frac{x}{2} + 5 \leq 6$ or $\frac{x}{3} + 2 \geq 4$

Write you final answer in interval notation.