5.2- Multiplication of Polynomials

$a^m \cdot a^n = a^{m+n}$

Example 1: Multiply and Simplify

a) $(-8x^4y^7)(5x^3y^2)$

$= -8 \cdot 5 \cdot x^4 \cdot x^3 \cdot y^7 \cdot y^2$

$= -40x^{4+3}y^{7+2}$

$= -40x^7y^9$

b) $(-3a^5bc^6)(-4a^2b^5c^8)$

$= -3(-4) \cdot a^5 \cdot a^2 \cdot b \cdot b^5 \cdot c^6 \cdot c^8$

$= 12a^{5+2}b^{1+5}c^{6+8}$

$= 12a^7b^6c^{14}$

c) $(6nm^8)(-n^2m^3)$

$= -6n^3m^{11}$

Multiplying Monomials and Binomials

Example 2: Multiply

a) $2t(3t-5)$

$= (2t)(3t) + (2t)(-5)$

$= 6t^2 - 10t$

b) $3a^2b(a^2-b^2)$

$= (3a^2b)(a^2) + (3a^2b)(-b^2)$

$= 3a^4b - 3a^2b^3$

c) $5x^2y^3(3x-4y^2)$

$= (5x^2y^3)(3x) + (5x^2y^3)(-4y^2)$

$= 15x^3y^3 - 20x^2y^5$

Example 3: Multiplying a Binomial and a Binomial

First Outer Inner Last

F O I L

$(y^3 - 5)(2y^3 + 4)$

	$2y^3$	$+4$
y^3	$2y^6$	$4y^3$
-5	$-10y^3$	-20

$= 2y^6 + 4y^3 - 10y^3 - 20$

$= 2y^6 - 6y^3 - 20$

$(a^2 - 2)(3a^2 + 5)$

$a^2 \cdot (3a^2) + a^2(5) + (-2)(3a^2) + (-2)(5)$

$= 3a^4 + 5a^2 - 6a^2 - 10$

$= 3a^4 - a^2 - 10$

$(x - 4)(x - 8)$

$= x(x-8) - 4(x-8)$

$= x(x) - 8x - 4x + (-4)(-8)$

$= x^2 - 8x - 4x + 32$

$= x^2 - 12x + 32$

$(2x + 3y)(x - 4y)$

$= 2x(x-4y) + 3y(x-4y)$

$= 2x(x) + (2x)(-4y) + (3y)(x) + 3y(-4y)$

$= 2x^2 - 8xy + 3xy - 12y^2$

$= 2x^2 - 5xy - 12y^2$

Example 4: Multiplying any two Polynomials

a) $(p + 2)(p^4 - 2p^3 + 3)$

	p^4	$-2p^3$	$+3$
p	$p(p^4) = p^5$	$p(-2p^3) = -2p^4$	$p(3) = 3p$
$+2$	$+2(p^4) = 2p^4$	$+2(-2p^3) = -4p^3$	$+2(3) = 6$

$= p^5 - 2p^4 + 3p + 2p^4 - 4p^3 + 6$

$= p^5 - 4p^3 + 3p + 6$

b) $(x+3)(x^3-5x-1)$

	x^3	$-5x$	-1
x	x^4	$-5x^2$	$-x$
3	$3x^3$	$-15x$	-3

$= x^4 - 5x^2 - x + 3x^3 - 15x - 3 = x^4 + 3x^3 - 5x^2 - 16x - 3$

c) $(5x^3+x-4)(-2x^2+3x+6)$

	$-2x^2$	$3x$	$+6$
$5x^3$	$-10x^5$	$15x^4$	$30x^3$
$+x$	$-2x^3$	$3x^2$	$6x$
-4	$8x^2$	$-12x$	-24

$= -10x^5 + 15x^4 + 30x^3 - 2x^3 + 3x^2 + 6x + 8x^2 - 12x - 24$

$= -10x^5 + 15x^4 + 28x^3 + 11x^2 - 6x - 24$

d) $(2x^2+8x-7)(x^2+x-4)$

	x^2	$+x$	-4
$2x^2$	$2x^4$	$2x^3$	$-8x^2$
$+8x$	$8x^3$	$8x^2$	$-32x$
-7	$-7x^2$	$-7x$	28

$= 2x^4 + 8x^3 + 2x^3 - 7x^2 + 8x^2 - 8x^2 - 7x - 32x + 28$

$= 2x^4 + 10x^3 - 7x^2 - 39x + 28$

e) $(t+2)(t-4)(t+5)$

$(t-4)(t+5) = t(t+5) - 4(t+5)$

$= t^2 + 5t - 4t - 20$

$= t^2 + t - 20$

$= (t+2)(t^2+t-20)$

	t^2	t	-20
t	t^3	t^2	$-20t$
2	$2t^2$	$2t$	-40

$= t^3 + 2t^2 + t^2 + 2t - 20t - 40 = t^3 + 3t^2 - 18t - 40$

Example 5: Squaring a binomial $(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^2 = a^2 + b^2$ ✗

a) $(y-5)^2 \equiv (a+b)^2$

$a = y$, $b = -5$

$= (y)^2 + 2(y)(-5) + (-5)^2$

$= y^2 - 10y + 25$

b) $(2x+3y)^2$

a b

$= (2x)^2 + 2(2x)(3y) + (3y)^2$

$= 4x^2 + 12xy + 9y^2$

c) $\left(\frac{1}{2}x - 3y^4\right)^2$

a b

$= \left(\frac{1}{2}x\right)^2 + 2\left(\frac{1}{2}x\right)(-3y^4) + (-3y^4)^2$

$= \frac{1}{4}x^2 - 3xy^4 + 9y^8$

Example 6: Products of Sum and Differences $(a+b)(a-b) = (a-b)(a+b) = a^2 - b^2$

a) $(t+5)(t-5) \equiv (a+b)(a-b)$

a b

$= (t)^2 - (5)^2$

$= t^2 - 25$

b) $(2xy^2 + 3x)(2xy^2 - 3x)$

a b

$= (2xy^2)^2 - (3x)^2$

$= 4x^2y^4 - 9x^2$

Example 7: Given $f(x) = x^2 - 4x + 5$, find and simplify each of the following

a) $f(a) + 3$

$= a^2 - 4a + 5 + 3 = a^2 - 4a + 8$

b) $f(a + 3)$

$= (a+3)^2 - 4(a+3) + 5$

$= a^2 + 2(a)(3) + (3)^2 - 4a - 12 + 5$

identity; distribute -4

$= a^2 + 6a + 9 - 4a - 12 + 5 = a^2 + 2a + 2$

c) $f(a + h) - f(a)$

$= (a+h)^2 - 4(a+h) + 5 - (a^2 - 4a + 5)$

$f(a+h)$; $f(a)$

$= a^2 + 2ah + h^2 - 4a - 4h + 5 - a^2 + 4a - 5$

identity; distribute -4; distribute -1

$= a^2 + 2ah + h^2 - 4a - 4h + 5 - a^2 + 4a - 5$

$= h^2 + 2ah - 4h$