Math -I 110 5.3 Notes

$(t+4)(t-5) = t^2 - t - 20$ → trinomial

" $(t+4)(t-5)$ → product of binomials; factor 1, factor 2

Factoring

Factoring is the reverse of multiplication.

To **factor** a polynomial is to find an equivalent expression that is the product of polynomials. An equivalent expression of this type is called a *factorization* of the polynomial.

Terms with Common Factors

When factoring a polynomial, we look for factors common to every term and then use the distributive law.

Example: Write an equivalent expression by factoring out the common factor

$10x^2 + 35$ $= 5 \cdot 2x^2 + 5 \cdot 7$ $= 5(2x^2 + 7)$	$2y^2 - 18y$ $= 2(y^2 - 9y)$ $= 2y\left(\frac{y^2}{y} - \frac{9y}{y}\right) = 2y(y-9)$ cannot factor any further $y\left(1 - \frac{9}{y}\right)$ not a Polynomial
$5x^2 - 15x + 5$ $= 5(x^2 - 3x + 1)$	$x^6 + 2x^4 - x^3$ $= x^3(x^3 + 2x - 1)$
$12x^4 - 30x^3 + 42x$ $= 6x\left(\frac{12x^4}{6x} - \frac{30x^3}{6x} + \frac{42x}{6x}\right)$ $= 6x(2x^3 - 5x^2 + 7)$	$16a^2b - 2ab - 9b$ $= b(16a^2 - 2a - 9)$
$15m^4n + 30m^5n^2 + 25m^3n^3$ $= 5m^3n\left(\frac{15m^4n}{5m^3n} + \frac{30m^5n^2}{5m^3n} + \frac{25m^3n^3}{5m^3n}\right)$ $= 5m^3n(3m + 6m^2n + 5n^2)$	$9x^3y^6z^2 - 12x^4y^4z^4 + 15x^2y^5z^3$ $= 3x^2y^4z^2(3xy^2 - 4x^2z^2 + 5yz)$

Factoring Completely

Polynomials that cannot be factored further are said to be **factored completely**. The factors in the resulting factorization are said to be **prime polynomials**.

When the leading coefficient is a negative number, we generally factor out a common factor with a negative coefficient.

Write an equivalent expression by factoring a factor with a negative coefficient

$-5x - 40$ $= -5(x + 8)$	$-16t^2 + 96$ $= -16(t^2 - 6)$
$-2x^2 + 12x + 40$ $= -2(x^2 - 6x - 20)$	$5 - 10y$ $= -5(-1 + 2y) = -5(2y - 1)$
$-p^3 - 2p^2 - 5p + 2$ $= -1(p^3 + 2p^2 + 5p - 2)$	$-a^5 - 5a^4 - 11a + 10$ $= -(a^5 + 5a^4 + 11a - 10)$

Factor by Grouping

$a(b-5) + c(b-5)$ $= (b-5)(a+c)$	$(x+7)(x-1) + (x+7)(x-2)$ $= (x+7)[(x-1) + (x-2)]$ $= (x+7)[x-1+x-2]$ $= (x+7)(2x-3)$
$5x^2(x-6) + 2(x-6)$ $= (x-6)(5x^2 + 2)$	$xy + xz + wy + wz$ $= x(y+z) + w(y+z)$ $= (y+z)(x+w)$

$y^3 - y^2 + 3y - 3 = y^3 - 3 - y^2 + 3y = (y^3 - 3) - y(y - 3)$ Not correct grouping

Factor by Grouping

$y^3 - y^2 + 3y - 3$	$t^3 + 6t^2 - 2t - 12$
$= y^2(y-1) + 3(y-1)$	$= t^2(t+6) - 2(t+6)$
$= (y-1)(y^2+3)$	$= (t+6)(t^2-2)$