5.8 APPLICATIONS OF POLYNOMIAL EQUATIONS

Polynomial Equations

When two polynomials are set equal, the result is called a **polynomial equation**. The **degree of a polynomial equation** is determined by the highest degree (or power) of any term in the equation. A **second-degree** polynomial equation in one variable is called a **quadratic equation**. Polynomial equations appear often in applications, so being able to solve them is an important skill. One common way to solve polynomial equations is by **factoring**.

$P(x) = 0$ or $P(x) = q(x)$

Understanding the Zero Product Property

When we multiply two or more numbers, the **product is zero** if any one of the numbers (factors) is zero. Conversely, if a product equals zero, then **at least one factor** must be zero.
This idea leads to an important rule used when solving equations by factoring.

$f(x) = g(x)$

The Principle of Zero Products

Quadratic eqns.

$3x^2+2=0$ or $3x^2+1=4-x^2$

$3x+4=0$

$\underbrace{3x+4}_{\text{deg } 1} = \underbrace{5-7x}_{\text{deg } 2}$

For any real numbers a and b:

- If $ab = 0$, then either $a = 0$ or $b = 0$
- If $a = 0$ or $b = 0$, then $ab = 0$

This principle allows us to break down equations into simpler parts and solve for each possible value of the variable.

$3x+4=0 \Rightarrow x = \frac{-4}{3}$

Steps for Solving by Factoring

- $x(3x+4)=0 \Rightarrow x=0$ or $3x+4=0 \Rightarrow x=\frac{-4}{3}$

1. Write the equation in **standard form**, with 0 on one side of the equation.
2. **Factor** the polynomial completely.
3. Apply the **zero product property** by setting each factor equal to 0.
4. Solve each resulting equation.
5. Check your solutions in the original equation.

- $x^2=0 \Rightarrow x \cdot x = 0 \Rightarrow x=0$

Objective: The principle of zero products

- $x^2=-1 \Rightarrow$ No solution

Solve:

$(x-2)(x-5)=0$ $x-2=0 \Rightarrow x=2$ or $x-5=0 \Rightarrow x=5$ $x=2,5$	$(x+3)(x+7)=0$ $x+3=0$ or $x+7=0$ $\Rightarrow x=-3$ or -7	$x^2+8x+7=0$ $\Rightarrow (x+1)(x+7)=0$ $\Rightarrow x=-1$ or -7	$15t^2-12t=0$ $\Rightarrow 3t(5t-4)=0 \Rightarrow 3t=0$ or $5t-4=0$ $\Rightarrow t=0$ or $\frac{4}{5}$
$x^2-3x-18=0$ $\Rightarrow (x+3)(x-6)=0$ $\Rightarrow x+3=0$ or $x-6=0$ $\Rightarrow x=-3$ or 6	$(3x-1)(4x-5)=0$ $3x-1=0 \Rightarrow 3x=1$ $4x-5=0$ $\quad \Rightarrow x=\frac{1}{3}$	$4x^2=10x$ $4x^2-10x=0$ $\Rightarrow 2x(2x-5)=0$ $\Rightarrow x=0$ or $5/2$	$n^2-81=0$ $\Rightarrow n^2-9^2=0$ $\Rightarrow (n-9)(n+9)=0$ $\Rightarrow n=9$ or -9

$\Rightarrow x=\frac{1}{3}$ or $\frac{5}{4}$

$-18 = -1 \times 18 = 1 \times (-18)$
$= -2 \times 9 = 2 \times (-9)$
$= -3 \times 6 = \boxed{3 \times (-6)}$

$4x-5=0$
$+5 \quad +5$
$\Rightarrow \frac{4x}{4} = \frac{5}{4}$
$\Rightarrow x = 5/4$

Example: Let $f(x) = x^2 + 14x + 50$. Find a such that $f(a) = 5$ $\Rightarrow a^2 + 14a + 50 = 5$

$45 = 1\times 45$
$= 3\times 15$
$= \boxed{5\times 9}$

$\Rightarrow a^2 + 14a + 50 - 5 = 0 \Rightarrow a^2 + 14a + 45 = 0$

$\Rightarrow (a+5)(a+9) = 0 \Rightarrow a+5=0$ or $a+9=0$

$\Rightarrow a = -5$ or -9

Example: $g(x) = 2x^2 - 15x$ Find a such that $g(a) = -7$

$\Rightarrow 2a^2 - 15a = -7 \Rightarrow 2a^2 - 15a + 7 = 0$

$14 = 1\times 14 = \boxed{-1\times -14}$

$\Rightarrow \underbrace{2a^2 - a}\ \underbrace{-14a + 7} = 0 \Rightarrow a(2a-1) - 7(2a-1) = 0$

$\Rightarrow (a-7)(2a-1) = 0 \Rightarrow a = 7$ or $a = \frac{1}{2}$

Example: If $f(x) = 12x^2 - 12x$ and $g(x) = 8x - 5$. Find all x-vales for which $f(x) = g(x)$

$12x^2 - 12x = 8x - 5 \Rightarrow 12x^2 - 20x + 5 = 0$
$-8x+5 \qquad -8x+5$

$60 = -1\times -60$
$= -2\times 30$
$= -3\times -20$
$= -4\times -15$
$= -5\times -12$
$= -6\times -10$

none sum to -20

$\Rightarrow$ Cannot be factored

$\Rightarrow$ Have to use the quadratic formula

Example; A rectangle is 3 inches longer than it is wide, and its area is 180 square inches. Find the length and the width.

w $\qquad$ l

$l = w+3$ $\qquad$ $lw = 180$
$w+3$

$-180 = -1\times 180$
$= -2\times 90$
$= -3\times 60$
$= -4\times 45$
$= -6\times 30$
$= -9\times 20$
$= -10\times 18$
$= \boxed{-12\times 15}$

$\Rightarrow (w+3)w = 180$

$\Rightarrow w^2 + 3w - 180 = 0$

$\Rightarrow (w-12)(w+15) = 0$

$\Rightarrow w = 12$ or $\boxed{-15}$ not Possible

$\Rightarrow w = 12$ inches
$\Rightarrow l = w+3 = 15$ inches

Example: A photo is 3 centimeters longer than it is wide, and its area is 108 square centimeters. Find the length and the width.

$l = w+3$ $\qquad$ $lw = 108$

$\Rightarrow w(w+3) = 108$

$\vdots$

$\Rightarrow$ what if we want cloth to hang equally on all sides?

$A = 60\cdot(40) = 2400$ | $\frac{84.9-60}{2} = \frac{24.9}{2} = 12.45$ along length | $\frac{56.6-40}{2} = 8.3$ along width

Example: A rectangular table is 60 inches long and 40 inches wide. A tablecloth that is twice the area of the table will be centered on the table. How far will the tablecloth hang down on each side?

Cloth has area $= 2(2400) = 4800 = lw$ $\Rightarrow$ l, w of the cloth

Assume $\frac{\text{length}}{\text{width}}$ of cloth $= \frac{60}{40} = \frac{3}{2} \Rightarrow \frac{l}{w} = \frac{3}{2} \Rightarrow \boxed{l = \frac{3}{2}w}$ $\rightarrow l = \frac{3}{2}(56.6) = 84.9$

$\Rightarrow \left(\frac{3}{2}w\right)w = 4800 \Rightarrow \frac{3}{2}w^2 = 4800 \Rightarrow w^2 = \frac{2}{3}(4800) \Rightarrow w^2 = 3200$

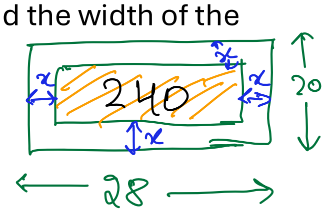
$\Rightarrow w = \sqrt{3200} = 56.6$

Suppose that a flare is launched upward with an initial velocity of 80ft sec from a height of 224 ft. Its height in feet, $h(t)$, after t seconds is given by $h(t) = -16t^2 + 80t + 224$How long will it take the flare to reach the ground

$\rightarrow h(t) = 0 \Rightarrow t$ for which $h(t) = 0$

$-16t^2 + 80t + 224 = 0$

A picture frame measures 28 in. by 20 in., and 240 square inches of picture shows. Find the width of the frame. dimensions of Picture $= (28-2x)\times(20-2x)$



$\Rightarrow$ Area $= 240 \Rightarrow (28-2x)(20-2x) = 240$

$\Rightarrow 560 \underbrace{-40x - 56x} + 4x^2 = 240 \Rightarrow 4x^2 - 96x + 560 - 240 = 0$

$\Rightarrow 4x^2 - 96x + 320 = 0 \Rightarrow 4(x^2 - 24x + 80) = 0 \Rightarrow 4(x-4)(x-20) = 0$

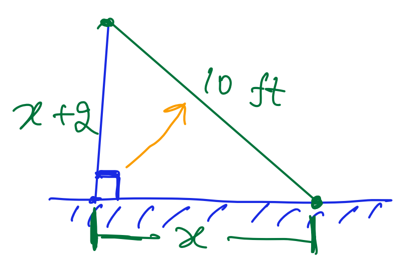
$\Rightarrow x = 4$ or $\cancel{20} \rightarrow 20-2x$ Cannot be $-$ve. $\Rightarrow$ width of frame $=$ 4 inches

A wire is stretched from the ground to the top of a pole. The wire is 10 ft long. The height of the pole is 2 ft greater than the distance x from the bottom of the pole to the bottom of the wire. Find the height of the pole and the distance x.

Pythagoras theorem :—

$\underbrace{(\text{Hypotenuse})}^2 = (\text{Perp.})^2 + (\text{Base})^2$

side facing 90° angle.



$10^2 = (x+2)^2 + x^2 \Rightarrow x^2 + 2(x)(2) + 2^2 + x^2 = 100$

$\Rightarrow 2x^2 + 4x + 4 = 100$

$\Rightarrow 2x^2 + 4x + 4 - 100 = 0$

$$\Rightarrow 2x^2 + 4x - 96 = 0 \Rightarrow 2(\underbrace{x^2 + 2x - 48}) = 0$$

$$\Rightarrow 2(x^2 + 8x - 6x - 48) = 0$$

$$\Rightarrow 2(x+8)(x-6) = 0$$

$$\Rightarrow x = \cancel{-8}, \quad x = 6$$

distance cannot be -ve

$$\Rightarrow x = 6 \Rightarrow \text{height of Pole} = x+2 = 6+2 = 8 \text{ ft.}$$

In-Class Quiz

1. Rationalize the denominator :

5 pts (a) $\dfrac{2\sqrt{3}}{\sqrt{5}}$

5 pts. (b) $\dfrac{3\sqrt{8}}{\sqrt{2}}$

2. Let $g(x) = x^2 - 4x$.

5 pts. Find x for which $g(x) = -4$

3. Let $f(x) = x^2 + x$ and $g(x) = x + 1$

5 pts. Find x for which $f(x) = g(x)$.