7.1- Radical Expressions and Functions

$2^2 = 2\cdot 2 = 4, \ (-2)^2 = 4$

Square Roots and Square Root Functions

EXAMPLE 1: Find the two square roots of 36

$6, -6$

A number x s.t. $x^2 = 36$

$\Rightarrow x = 6, -6$

EXAMPLE 2: Find the two square roots of 49

7 and -7

$(7)^2 = 49$

$(-7)^2 = 49$

Positive answers.

PRINCIPAL SQUARE ROOT $\sqrt{x}$

EXAMPLE 3: Simplify each of the following

a) $\sqrt{25}$

$= 5$

b) $\sqrt{\frac{25}{64}} = \frac{\sqrt{25}}{\sqrt{64}}$

$= \frac{5}{8}$

c) $-\sqrt{64}$

$= -8$

d) $\sqrt{0.0049}$ 4 Places

$= \sqrt{\frac{49}{10000}}$

$= \frac{7}{100} = 0.07$

0.07 2 Places

• $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\sqrt{0.49} = 0.7$

EXAMPLE 4: For each function, find the indicated function value

a) $f(x) = \sqrt{3x - 2}; f(1)$

$f(1) = \sqrt{3(1)-2}$

$= \sqrt{1} = 1$

b) $g(x) = \sqrt{6x + 4}\ ; g(3)$

$g(3) = \sqrt{6(3)+4}$

$= \sqrt{18+4} = \sqrt{22}$

$= 4.69$

$\boxed{(\sqrt{a})^2 = a, \ a \text{ is +ve}}$

Expressions of the form $\sqrt{a^2}$

EXAMPLE 5: Evaluate $\sqrt{a^2}$ for each of the following values

a) 5

$\sqrt{5^2} = \sqrt{25}$

$= 5$

b) 0

$\sqrt{0^2} = \sqrt{0}$

$= 0$

c) -5

$\sqrt{(-5)^2} = \sqrt{25} = 5$

- $\boxed{\sqrt{a^2} = |a|}$ (not equal to a)
- $\boxed{\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}}$

EXAMPLE 6: Simplify each expression. Assume that the variable can represent any real number

a) $\sqrt{36t^2}$

$= \sqrt{36}\sqrt{t^2}$

$= 6|t|$

b) $\sqrt{(\underbrace{x+1}_{a=x+1})^2}$

$= |x+1|$

c) $\sqrt{x^2 - 8x + 16} = \sqrt{(x-4)^2}$

$x^2 - \underbrace{8x}_{2(x)} + \underbrace{16}_{(-4)^2} = (x-4)^2$

$a = x$

$b = -4$

$|x-4|$

$\boxed{a^2 + 2ab + b^2 = (a+b)^2}$

d) $\sqrt{t^6}$

$= \sqrt{(t^3)^2}$

$= |t^3|$

e) $\sqrt{z^8}$

$= \sqrt{(z^4)^2}$

$= |z^4| = z^4$

$a^2+2ab+b^2 = (a+b)^2$

EXAMPLE 7: Simplify each expression. Assume that the expressions being square are nonnegative. Thus absolute value notation is not necessary.

a) $\sqrt{y^2}$

$= y$

b) $\sqrt{a^{10}}$

$= a^5$

c) $\sqrt{9x^2 - 6x + 1}$

$(3x)^2$

$a = 3x, \quad b = -1$

$\frac{-6x}{2(3x)} = -1 = b$

$\sqrt{(3x-1)^2}$

$= 3x - 1$

Cube roots $\boxed{x^3 = \text{given number}}$

$2^3 = 8$

$(-2)^3 = -8$

$\sqrt[3]{8} = 2$

$\sqrt[3]{-8} = -2$

EXAMPLE 8: For each function, find the indicated function value

a) $f(x) = \sqrt[3]{x}; f(125)$

$f(125) = \sqrt[3]{125}$

$= 5$

b) $g(x) = \sqrt[3]{x-1}\ ; g(-26)$

$g(-26) = \sqrt[3]{-26-1}$

$= \sqrt[3]{-27} = -3$

EXAMPLE 9: Simplify $\sqrt[3]{-8y^3}$

$= \sqrt[3]{-8} \cdot \sqrt[3]{y^3} = -2y$

$\boxed{\bullet\ \sqrt[3]{ab} = \sqrt[3]{a}\ \sqrt[3]{b}}$

ODD AND EVEN NTH ROOTS If $x^n = \text{given number}$ then $x = \sqrt[n]{\text{given number}}$

EXAMPLE 10: Simplify each expression

a) $\sqrt[5]{32}$

$= 2$

b) $\sqrt[5]{-32}$

$= -2$

c) $-\sqrt[5]{32}$

$= -2$

$2^5 = 32$

d) $-\sqrt[5]{-32}$

$-(-2)$
$=2$

e) $\sqrt[7]{x^7}$

$= x$

f) $\sqrt[9]{(t-1)^9}$

$= t-1$

(*) $\sqrt[8]{x^8} = |x|$

EXAMPLE 11: Simplify each expression, if possible. Assume that variables can represent any number.

a) $\sqrt[4]{81}$

$= 3$

b) $-\sqrt[4]{81}$

$= -3$

c) $\sqrt[4]{81x^4}$

$= 3|x|$

d) $\sqrt[6]{(y+7)^6}$

$= |y+7|$

EXAMPLE 12: Determine the domain of g if $g(x) = \sqrt[6]{7-3x}$

$7-3x \geq 0$

$\Rightarrow \underset{-7}{7}-3x \geq \underset{-7}{0} \Rightarrow -3x \geq -7$

$\Rightarrow \frac{-3x}{-3} \leq \frac{-7}{-3} \Rightarrow x \leq \frac{7}{3}$

$\Rightarrow$ Domain of $g = (-\infty, \frac{7}{3}]$