7.2- Rational Numbers as Exponents

Rational Exponents

$\sqrt{x} = x^{1/2}$

$\sqrt[3]{x} = x^{1/3}$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$a^{\frac{1}{n}}$ means the nth root of a. When *a* is nonnegative, *n* can be any natural number greater than 1. When *a* is negative, *n* can be any odd natural number greater than 1.

THE DENOMINATOR OF THE EXPONENET IS THE INDEX OF THE RADICAL EXPRESSION

EXAMPLE 1: Write an equivalent expression using radical notation and, if possible simplify

a) $16^{\frac{1}{2}}$

$= \sqrt{16}$

$= 4$

b) $(-8)^{\frac{1}{3}}$

$= \sqrt[3]{-8}$

$= -2$

c) $(abc)^{\frac{1}{5}}$

$= \sqrt[5]{abc}$

d) $(25x^{16})^{\frac{1}{2}}$)

$= \sqrt{25x^{16}}$

$= 5x^4$

EXAMPLE 2: Write an equivalent expression using exponential notation

a) $\sqrt[5]{7ab}$

$= (7ab)^{\frac{1}{5}}$

b) $\sqrt[7]{\frac{x^3y}{4}}$

$= \left(\frac{x^3y}{4}\right)^{\frac{1}{7}}$

c) $\sqrt{5x}$

$= (5x)^{1/2}$

For any natural numbers m and n ($n \neq 1$) and any real number a for which $\sqrt[n]{a}$ exists,

$a^{\frac{m}{n}}$ means $(\sqrt[n]{a})^m$, or $\sqrt[n]{a^m}$

EXAMPLE 3: Write an equivalent expression using radical notation and simplify.

a) $27^{\frac{2}{3}}$

$= (\sqrt[3]{27})^2$

$= 3^2$

$= 9$

b) $25^{\frac{3}{2}}$

$= (\sqrt{25})^3$

$= 5^3 = 125$

EXAMPLE 4: Write an equivalent expression using exponential notation

a) $\sqrt[3]{9^4}$

$= 9^{\frac{4}{3}}$

b) $(\sqrt[4]{7xy})^5$

$= (7xy)^{5/4}$

$a^{-m} = \frac{1}{a^m}$

NEGATIVE RATIONAL EXPONENTS

Negative Rational Exponents

For any rational number *m/n* and any nonzero real number *a* for which $a^{\frac{m}{n}}$ exists,

$a^{-\frac{m}{n}}$ means $\frac{1}{a^{\frac{m}{n}}}$.

Caution!
A negative exponent does not indicate that the expression in which it appears is negative: $a^{-1} \neq -a$.

$a^{-1} \to \frac{1}{a}$

EXAMPLE 5: Write an equivalent expression with positive exponents and, if possible, simplify

a) $9^{-\frac{1}{2}}$ $= \frac{1}{9^{1/2}}$ $= \frac{1}{3}$	b) $(5xy)^{-\frac{4}{5}}$ $= \frac{1}{(5xy)^{4/5}}$	c) $64^{-\frac{2}{3}}$ $= \frac{1}{64^{2/3}} = \frac{1}{(\sqrt[3]{64})^2}$ $= \frac{1}{4^2} = \frac{1}{16}$
d) $4x^{-\frac{2}{3}}y^{\frac{1}{5}}$ $= \frac{4y^{1/5}}{x^{2/3}}$	e) $\left(\frac{3r}{7s}\right)^{-\frac{5}{2}} = \frac{(3r)^{-5/2}}{(7s)^{-5/2}}$ $= \frac{1}{\left(\frac{3r}{7s}\right)^{5/2}}$ $= \frac{(7s)^{5/2}}{(3r)^{5/2}}$	• $\left(\frac{a}{b}\right)^{m/n} = \frac{a^{m/n}}{b^{m/n}}$

LAW OF EXPONENTS
For any real numbers *a* and *b* and any rational exponents *m* and *n* for which a^m, a^n, and b^m are defined:

1. $a^m \cdot a^n = a^{m+n}$
   When multiplying, add exponents if the bases are the same.
2. $\frac{a^m}{a^n} = a^{m-n}$
   When dividing, subtract exponents if the bases are the same. (Assume *a ≠ 0*.)
3. $(a^m)^n = a^{m \cdot n}$
   To raise a power to a power, multiply the exponents.
4. $(ab)^m = a^m b^m$
   To raise a product to a power, raise each factor to the power and multiply.

$\frac{1}{4} - \frac{1}{2} = \frac{1}{4} - \frac{2}{4} = \frac{1-2}{4}$
$= -\frac{1}{4}$

EXAMPLE 6: Use the laws of exponents to simplify

a) $3^{\frac{1}{5}} \cdot 3^{\frac{3}{5}}$

$3^{\frac{1}{5}+\frac{3}{5}} = 3^{4/5}$

b) $\frac{a^{\frac{1}{4}}}{a^{\frac{1}{2}}}$

$= a^{\frac{1}{4}-\frac{1}{2}} = a^{-1/4}$

$= \frac{1}{a^{1/4}}$

c) $\left(7.2^{\frac{2}{3}}\right)^{\frac{3}{4}}$

$= 7^{3/4} \cdot \left(2^{2/3}\right)^{3/4}$

$= 7^{3/4} \cdot 2^{\frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{4}}$

$= 7^{3/4} \cdot 2^{1/2}$

d) $\left(a^{-\frac{1}{3}} b^{\frac{2}{5}}\right)^{\frac{1}{2}}$

$= \left(a^{-1/3}\right)^{1/2} \cdot \left(b^{2/5}\right)^{1/2}$

$= a^{-1/6} \cdot b^{1/5} = \frac{b^{1/5}}{a^{1/6}}$

SIMPLIFYING RADICAL EXRPESSIONS

1. Convert radical expressions to exponential expressions.
2. Use arithmetic and the laws of exponents to simplify.
3. Convert back to radical notation as needed.

EXAMPLE 7: Use rational exponents to simplify. DO not use exponents that are fraction in the final answer

a) $\sqrt[6]{(5x)^3}$

$= (5x)^{3/6} = (5x)^{1/2}$

$= \sqrt{5x}$

b) $\sqrt[5]{t^{20}}$

$= t^{20/5} = t^4$

c) $\left(\sqrt[3]{ab^2c}\right)^{12}$

$= (ab^2c)^{12/3}$

$= (ab^2c)^4 = a^4 b^8 c^4$

d) $\sqrt{\sqrt[3]{x}}$

$= \sqrt{x^{1/3}} = \left(x^{1/3}\right)^{1/2}$

$= x^{1/6} = \sqrt[6]{x}$

$= \left(\sqrt[3]{x}\right)^{1/2}$

$= \left(x^{1/3}\right)^{1/2} = x^{1/6} = \sqrt[6]{x}$

H.W.

(1) $\sqrt[4]{\sqrt[3]{64}}$

$= \sqrt[4]{4} = 4^{1/4} = (2^2)^{1/4} = 2^{2 \cdot \frac{1}{4}} = 2^{1/2}$

$= \sqrt{2}$