MATH-I 110 7.3 NOTES

Objective: Multiply radical expressions

The Product Rule for Radicals

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$,

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

(The product of two *nth* roots is the *nth* root of the product of the two radicands.)

Example: Multiply

$\sqrt{3} \cdot \sqrt{10}$ $= \sqrt{3 \cdot 10}$ $= \sqrt{30}$	$\sqrt{6} \cdot \sqrt{5}$ $= \sqrt{6 \cdot 5}$ $= \sqrt{30}$	$\sqrt[3]{7} \cdot \sqrt[3]{5}$ $= \sqrt[3]{7 \cdot 5}$ $= \sqrt[3]{35}$	$\sqrt[3]{12} \cdot \sqrt[3]{3}$ $= \sqrt[3]{12 \cdot 3}$ $= \sqrt[3]{36}$
$\sqrt{12} \cdot \sqrt{3}$ $= \sqrt{36}$ $= 6$	$\sqrt{6} \cdot \sqrt{9}$ $= \sqrt{54}$	$\sqrt[4]{4} \cdot \sqrt[4]{10}$ $= \sqrt[4]{4 \cdot 10}$ $= \sqrt[4]{40}$	$\sqrt{2x} \cdot \sqrt{13y}$ $= \sqrt{2x(13y)}$ $= \sqrt{26xy}$
$\sqrt{5a} \cdot \sqrt{6b}$ $= \sqrt{30ab}$	$\sqrt[5]{3y^3} \cdot \sqrt[5]{10y}$ $= \sqrt[5]{30y^4}$	$\sqrt{y-b} \cdot \sqrt{y+b}$ $= \sqrt{(y-b)(y+b)}$ $= \sqrt{y^2-b^2}$	$\sqrt{a-x} \cdot \sqrt{x+a}$ $= \sqrt{(a-x)(x+a)}$ $= \sqrt{(a-x)(a+x)}$ $= \sqrt{a^2-x^2}$

$\boxed{\sqrt{2}\ \sqrt[3]{6} \neq \sqrt{12} \text{ or } \sqrt[3]{12}}$

$\boxed{2^{1/2}\ 6^{1/3} \neq 2^{\frac{1}{2}+\frac{1}{3}} \text{ or } 6^{\frac{1}{2}+\frac{1}{3}}}$

Objective: Simplify by Factoring $\boxed{\sqrt{512}} = \sqrt{4\cdot128} = 2\sqrt{128} = 2\sqrt{4\cdot32}$

$= 2\cdot2\sqrt{32} = 4\sqrt{16\cdot2}$

$= 4\cdot4\sqrt{2}$

$= 16\sqrt{2}$

Simplify. Assume that no radicands were formed by raising negative numbers to even powers

$\sqrt{12}$ $=\sqrt{4\times3}$ $=\sqrt{4}\,\sqrt{3}$ $=2\sqrt{3}$	$\sqrt{27}$ $=\sqrt{9\times3}$ $=\sqrt{9}\,\sqrt{3}$ $=3\sqrt{3}$	$\sqrt{120}$ $=\sqrt{4\times30}$ $=\sqrt{4}\,\sqrt{30}$ $=2\sqrt{30}$	$\sqrt[3]{27ab^6}$ $=\sqrt[3]{27}\,\sqrt[3]{a}\,\sqrt[3]{b^6}$ $=3\sqrt[3]{a}\,\sqrt[3]{(b^2)^3}$ $=3b^2\sqrt[3]{a}$
$\sqrt{300}$ $\sqrt{3\times100}$ $=10\sqrt{3}$ $=\sqrt{4\times75}$ $=\sqrt{4}\,\sqrt{75}=2\sqrt{75}$ $=2\sqrt{25\times3}$ $=2\cdot5\sqrt{3}=10\sqrt{3}$	$\sqrt[3]{8x^9}$ $=\sqrt[3]{8}\,\sqrt[3]{x^9}$ $=2\sqrt[3]{(x^3)^3}$ $=2x^3$	$\sqrt{350}$ $=\sqrt{5\cdot70}$ $=\sqrt{\underbrace{5\cdot5}\cdot14}$ $=\sqrt{5^2}\,\sqrt{14}$ $=5\sqrt{14}$	$\sqrt{175y^8}$ $=\sqrt{175}\,\sqrt{y^8}$ $=\sqrt{5\cdot35}\,\sqrt{(y^4)^2}$ $=\sqrt{5\cdot5\cdot7}\;y^4$ $=5y^4\sqrt{7}=5\sqrt{7}y^4$
$\sqrt{45}$ $=\sqrt{9\cdot5}$ $=3\sqrt{5}$	$\sqrt{75y^5}$ $=\sqrt{25\cdot3}\,\sqrt{y\cdot y^4}$ $=5\sqrt{3}\cdot\sqrt{y}\cdot\sqrt{y^4}$ $=5\sqrt{3}\cdot\sqrt{y}\cdot y^2$ $=5y^2\sqrt{3y}$	$\sqrt{36a^4b^4}$ $=\sqrt{36}\,\sqrt{a^4}\,\sqrt{b^4}$ $=6a^2b^2$	$\sqrt[3]{8x^3y^2}$ $=\sqrt[3]{8}\,\sqrt[3]{x^3}\,\sqrt[3]{y^2}$ $=2x\sqrt[3]{y^2}$
$\sqrt[3]{-16x^6}$ $=\sqrt[3]{-16}\,\sqrt[3]{x^6}$ $=\sqrt[3]{-1}\,\sqrt[3]{\underbrace{8}\cdot2}\,\sqrt[3]{x^6}$ $=-2\sqrt[3]{2}\,x^2=-2x^2\sqrt[3]{2}$		$\sqrt[3]{(-32a^6)}$ $=\sqrt[3]{-1\cdot8\cdot4\cdot a^6}$ $=\sqrt[3]{-1}\,\sqrt[3]{8}\,\sqrt[3]{4}\,\sqrt[3]{a^6}$ $=-2a^2\sqrt[3]{4}$	

$(a+b)^2 = a^2+2ab+b^2$, $(a-b)^2 = a^2-2ab+b^2$, (a, b are +ve)

Find a simplified form of $f(x)$. Assume that x can be any real number.

$f(x) = \sqrt[3]{40x^6}$ $f(x) = \sqrt[3]{8\cdot 5}\,\sqrt[3]{x^6}$ $= \sqrt[3]{8}\,\sqrt[3]{5}\,\sqrt[3]{(x^2)^3}$ $= 2x^2\sqrt[3]{5} = 2\sqrt[3]{5}\,x^2$	$f(x) = \sqrt{81(x-1)^2}$ $f(x) = \sqrt{81}\,\sqrt{(x-1)^2}$ $= 9\lvert x-1\rvert$
$f(x) = \sqrt[3]{(27x^5)}$ $f(x) = \sqrt[3]{27}\,\sqrt[3]{x^5} = 3\sqrt[3]{x^5}$ $= 3\sqrt[3]{x^3\cdot x^2} = 3\sqrt[3]{x^3}\,\sqrt[3]{x^2}$ $= 3x\sqrt[3]{x^2}$	$f(x) = \sqrt{5x^2 - 10x + 5}$ $f(x) = \sqrt{5(x^2-2x+1)}$ $(x)^2+2(x)(-1)+(-1)^2$ $= \sqrt{5(x-1)^2}$ $= \sqrt{5}\,\lvert x-1\rvert$
$f(x) = \sqrt{49(x-3)^2}$ $f(x) = \sqrt{49}\,\sqrt{(x-3)^2}$ $= 7\lvert x-3\rvert$	$f(x) = \sqrt{2x^2 + 8x + 8}$ $f(x) = \sqrt{2(x^2+4x+4)}$ $= \sqrt{2}\,\lvert x+2\rvert$

Simplify. Assume that no radicands were formed by raising negative numbers to even powers

$\sqrt{a^{10}b^{11}}$ $= \sqrt{a^{10}}\,\sqrt{b^{10}\cdot b}$ $= a^5b^5\sqrt{b}$	$\sqrt[4]{x^5y^6z^{10}}$ $= \sqrt[4]{x^4\cdot x}\,\sqrt[4]{y^4\cdot y^2}\,\sqrt[4]{z^8\cdot z^2}$ $= x\sqrt[4]{x}\;y\sqrt[4]{y^2}\;z^2\sqrt[4]{z^2}$ $= xyz^2\sqrt[4]{xy^2z^2}$	$\sqrt[4]{16x^5y^{11}}$ $= \sqrt[4]{2^4x^4\cdot x\,y^8\cdot y^3}$ $= 2xy^2\sqrt[4]{xy^3}$	$\sqrt[5]{x^{13}y^8z^{17}}$ $= \sqrt[5]{x^{10}x^3y^5y^3z^{15}\cdot z^2}$ $= x^2yz^3\sqrt[5]{x^3y^3z^2}$
$\sqrt{x^8y^7}$ $= \sqrt{x^8\cdot y^6\cdot y}$ $= x^4y^3\sqrt{y}$	$\sqrt[3]{a^6b^7c^{13}}$ $= \sqrt[3]{a^6b^6b\,c^{12}c}$ $= a^2b^2c^4\sqrt[3]{bc}$	$\sqrt[5]{-32a^7b^{11}}$ $= \sqrt[5]{(-1)^5 2^5 a^5\cdot a^2 b^{10}\cdot b}$ $= -2ab^2\sqrt[5]{a^2b}$	$\sqrt[3]{80a^{14}}$ $= \sqrt[3]{2^3\cdot 10a^{12}a^2}$ $= 2a^4\sqrt[3]{10a^2}$

Objective: Multiplying and simplifying

$\sqrt{5}\cdot\sqrt{10}$ $=\sqrt{50}$ $=\sqrt{25}\sqrt{2}$ $=5\sqrt{2}$	$\sqrt{2}\cdot\sqrt{6}$	$\sqrt{6}\cdot\sqrt{33}$	$\sqrt{10}\cdot\sqrt{35}$	$\sqrt[3]{9}\cdot\sqrt[3]{3}$
$\sqrt[3]{2}\cdot\sqrt[3]{4}$ $=\sqrt[3]{8}$ $=2$	$\sqrt[5]{24y^5}\cdot\sqrt[5]{24y^5}$ $=\sqrt[5]{24\cdot 24y^{10}}$ $=\sqrt[5]{3\cdot 2^3\cdot 3\cdot 2^3y^{10}}$ $=\sqrt[5]{2^6\cdot 9\cdot y^{10}}$ $=\sqrt[5]{2^5\cdot 2\cdot 9\cdot y^{10}}=\boxed{2y^2\sqrt[5]{18}}$	$\sqrt{120t^9}\cdot\sqrt{120t^9}$ $\sqrt{120^2t^{18}}$ $=120\lvert t^9\rvert$	$\sqrt[3]{5a^2}\cdot\sqrt[3]{2a}$	$3\sqrt{2x^5}\cdot 4\sqrt{10x^2}$

$3\sqrt{5a^7}\cdot 2\sqrt{15a^3}$ $3\cdot 2\cdot\sqrt{5a^7\cdot 15a^3}$ $6\sqrt{75\cdot a^{10}}$ $6\sqrt{25\cdot 3\cdot a^{10}}$ $=6\cdot 5a^5\sqrt{3}$ $=30\sqrt{3}a^5$	$\sqrt[3]{(x^2y^4)}\cdot\sqrt[3]{(x^2y^6)}$ $=\sqrt[3]{x^4y^{10}}$ $=\sqrt[3]{x^3\cdot x\, y^9\cdot y}$ $=xy^3\sqrt[3]{xy}$	$\sqrt[3]{(t+4)^5}\cdot\sqrt[3]{t+4}$ $=\sqrt[3]{(t+4)^{5+1}}$ $=\sqrt[3]{(t+4)^6}$ $=(t+4)^2$	$\sqrt[4]{20a^3b^7}\cdot\sqrt[4]{4a^2b^5}$ $=\sqrt[4]{80a^5b^{12}}$ $=\sqrt[4]{2^4\cdot 5a^4\cdot a\, b^{12}}$ $=2ab^3\sqrt[4]{5a}$