

7.4 Dividing Radical Expressions

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Dividing and Simplifying

Simplify by taking the roots of the numerator and the denominator. Assume all variables represent positive numbers.

$\sqrt{\frac{49}{100}}$ $= \frac{\sqrt{49}}{\sqrt{100}} = \frac{7}{10}$	$\sqrt{\frac{81}{25}}$	$\sqrt[3]{\frac{125}{8}}$	$\sqrt[3]{\left(\frac{1000}{27}\right)}$ $= \frac{\sqrt[3]{1000}}{\sqrt[3]{27}} = \frac{10}{3}$
$\sqrt{\frac{121}{t^2}}$ $= \frac{\sqrt{121}}{\sqrt{t^2}} = \frac{11}{t}$	$\sqrt{\frac{144}{p^2}}$	$\sqrt{\frac{36x^3}{x^4}}$ $= \sqrt{\frac{6^2 \cancel{x^2} \cdot x}{\cancel{x^4}}}$ $= \frac{6x}{x^2} \sqrt{x} = \frac{6}{x} \sqrt{x}$	$\sqrt[3]{\frac{25a^5}{8b^3}}$ $= \sqrt[3]{\frac{25 \cdot \cancel{a^3} \cdot a^2}{2^3 \cdot \cancel{b^3}}}$ $= \frac{a}{2b} \sqrt[3]{25a^2} = \frac{a \sqrt[3]{25a^2}}{2b}$
$\sqrt[3]{\left(\frac{27a^4}{8b^3}\right)}$	$\sqrt[3]{\left(\frac{64x^7}{216y^6}\right)}$ $= \sqrt[3]{\frac{4^3 \cdot \cancel{x^6} \cdot x}{6^3 \cdot \cancel{y^6}}} = \frac{4x^2}{6y^2} \sqrt[3]{x}$ $= \frac{2x^2 \sqrt[3]{x}}{3y^2}$	$\sqrt{\frac{324a^4}{2b^4c^8}}$ $= \frac{18a^2}{b^2c^4\sqrt{2}}$	
$\sqrt[4]{\frac{a^5b^8}{c^{10}}}$ $= \sqrt[4]{\frac{\cancel{a^4} \cdot a \cdot \cancel{b^8}}{\cancel{c^8} \cdot c^2}} = \frac{ab^2\sqrt[4]{a}}{c^2\sqrt[4]{c^2}}$	$\sqrt[4]{\frac{x^9y^{12}}{z^6}}$ $= \frac{x^2y^3\sqrt[4]{x}}{z\sqrt[4]{z^2}}$	$\sqrt[5]{\frac{32x^6}{y^{11}}}$ $= \sqrt[5]{\frac{2^5 \cdot \cancel{x^5} \cdot x}{\cancel{y^{10}} \cdot y}} = \frac{2x\sqrt[5]{x}}{y^2\sqrt[5]{y}}$	

$\sqrt[5]{\frac{243a^9}{b^{13}}}$ $= \frac{3a \sqrt[5]{a^4}}{b^2 \sqrt[5]{b^3}}$	$\sqrt[6]{\frac{x^6 y^8}{z^{15}}}$ $= \frac{\sqrt[6]{x^6 y^6 \cdot y^2}}{\sqrt[6]{z^{12} \cdot z^3}} = \frac{xy \sqrt[6]{y^2}}{z^2 \sqrt[6]{z^3}}$	$\sqrt[6]{\frac{a^9 b^{12}}{c^{13}}}$ $= \frac{a b^2 \sqrt[6]{a^3}}{c^2 \sqrt[6]{c}}$
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Divide and, if possible, simplify. Assume that all variables represent positive numbers.

$\frac{\sqrt{18y}}{\sqrt{2y}}$ $= \sqrt{\frac{18y}{2y}} = \sqrt{9}$ $= 3$	$\frac{\sqrt{700x}}{\sqrt{7x}}$ $= \sqrt{\frac{700x}{7x}}$ $= \sqrt{100}$ $= 10$	$\frac{\sqrt[3]{26}}{\sqrt[3]{13}}$ $= \sqrt[3]{\frac{26}{13}}$ $= \sqrt[3]{2}$	$\frac{\sqrt[3]{35}}{\sqrt[3]{5}}$ $= \sqrt[3]{\frac{35}{5}}$ $= \sqrt[3]{7}$
$\frac{\sqrt{40xy^3}}{\sqrt{8x}}$ $= \sqrt{\frac{40xy^3}{8x}}$ $= \sqrt{5y^3} = \sqrt{5 \cdot y^2 \cdot y}$ $= y \sqrt{5y}$	$\frac{\sqrt{56ab^3}}{\sqrt{7a}}$ $= 2b \sqrt{2b}$	$\frac{\sqrt[3]{96a^4 b^2}}{\sqrt[3]{12a^2 b}}$ $= \sqrt[3]{\frac{96a^4 b^2}{12a^2 b}}$ $= \sqrt[3]{8a^2 b}$ $= 2 \sqrt[3]{a^2 b}$	$\frac{\sqrt[3]{189x^5 y^7}}{\sqrt[3]{7x^2 y^2}}$ $= \sqrt[3]{\frac{189x^5 y^7}{7x^2 y^2}}$ $= 3xy \sqrt[3]{y^2}$

Rationalizing Denominators or Numerators with One Term

Rationalize each denominator. Assume that all variables represent positive numbers.

get rid of radicals
in
either numerator or in denominator

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$$

Quiz 8

① Simplify radicals and write your answers in terms of positive exponents:

(a) $\frac{\sqrt[3]{2} \sqrt[3]{8}}{\sqrt[3]{2^2}} = \frac{2^{\frac{1}{3}} 2^1}{2^{\frac{2}{3}}} = 2^{\frac{1}{3} + 1 - \frac{2}{3}} = 2^{\frac{2}{3}}$

(b) $\frac{\sqrt{25} \sqrt{7}}{\sqrt{343}} = 5 \sqrt{\frac{7}{343}} = \frac{5}{7}$

② Simplify the exponents and write your answer in terms of radicals:

(a) $\frac{2^{\frac{1}{3}} 2^{\frac{1}{4}}}{2^{\frac{1}{2}}} = 2^{\frac{1}{3} + \frac{1}{4} - \frac{1}{2}} = 2^{\frac{5}{12} - \frac{6}{12}} = 2^{-\frac{1}{12}} = \frac{1}{2^{\frac{1}{12}}} = \frac{1}{\sqrt[12]{2}}$

(b) $5^{\frac{1}{2}} 5^{\frac{1}{3}} = 5^{\frac{1}{2} + \frac{1}{3}} = 5^{\frac{5}{6}} = \sqrt[6]{5^5} = \sqrt[6]{3125}$

Rationalize denominators.

$\frac{\sqrt{\frac{2}{5}}}{\frac{\sqrt{10}}{5}}$ $= \frac{\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{2} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$	$\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{2}$	$\frac{\frac{2\sqrt{5}}{7\sqrt{3}}}{\frac{2\sqrt{5}}{7\sqrt{3}}} = \frac{2\sqrt{5}}{7\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{15}}{7 \cdot 3}$ $= \frac{2\sqrt{15}}{21}$
$\frac{\frac{3\sqrt{5}}{2\sqrt{7}}}{\frac{3\sqrt{5}}{2\sqrt{7}}} = \frac{3\sqrt{35}}{14}$	$\frac{\sqrt[3]{\frac{5}{4}}}{\sqrt[3]{\frac{5}{4}}} = \frac{\sqrt[3]{5}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{10}}{\sqrt[3]{8}}$	$\frac{\sqrt[3]{\frac{2}{9}}}{\sqrt[3]{\frac{2}{9}}} = \frac{\sqrt[3]{2}}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{\sqrt[3]{6}}{\sqrt[3]{27}} = \frac{\sqrt[3]{6}}{3}$

$$= \frac{\sqrt[3]{10}}{2}$$