Quadratic formula

Math 110- 8.1

Solving equations with 1 term

$\boxed{x^2 = 17 \Rightarrow x = \pm\sqrt{17}}$

1. $4x^2 = 100$ $\frac{}{4} \quad \frac{}{4}$ $\Rightarrow x^2 = 25$ $\Rightarrow x = 5$ or -5	2. $x^2 = 16$ $\Rightarrow x = \pm 4$ $= 4$ or -4	3. $x^2 = 36$ $\Rightarrow x = 6$ or -6
4. $x^2 = 50$ $\Rightarrow x = \pm\sqrt{50}$ $= \pm\sqrt{25\times 2}$ $= \pm\sqrt{25}\sqrt{2}$ $\Rightarrow x = 5\sqrt{2}$ or $-5\sqrt{2}$	5. $4x^2 + 81 = 0$ $\Rightarrow 4x^2 = -81$ $\Rightarrow x^2 = -\frac{81}{4} \Rightarrow$ No such x $\left(-\frac{9}{2}\right)^2 = \frac{81}{4}$ $\Rightarrow$ No solutions	6. $(x+1)^2 = 100$ $\Rightarrow x+1 = 10$ or $x+1 = -10$ $\Rightarrow x = 9$ or -11
7. $(x+5)^2 = 9$ $\Rightarrow x+5 = 3$ or $x+5 = -3$ $\Rightarrow x = -2$ or $x = -8$	8. $(y-4)^2 = 18$ $\Rightarrow y-4 = \sqrt{18} = 3\sqrt{2}$ or $y-4 = -\sqrt{18} = -3\sqrt{2}$ $\Rightarrow y = 4+3\sqrt{2}$ or $4-3\sqrt{2}$	9. $3y^2 = 108$ $\Rightarrow y = 6$ or -6

Imaginary Numbers

→ Use when you are taking the square root of a negative number.

$$i = \sqrt{-1}$$

ex. $\sqrt{-16} = \sqrt{16} \cdot \sqrt{-1} = \sqrt{16}\, i = \boxed{4i}$

Solving by taking square roots

$$(x-3)^2 - 81 = 0$$

$$(x-3)^2 = 81$$

$$\sqrt{(x-3)^2} = \pm\sqrt{81}$$

$$x - 3 = \pm 9$$

$$x = \pm 9 + 3$$

$9x^2 + 16 = 0$

↓

$x^2 = -\frac{16}{9}$

⇒ No solutions

$16x^2 + 25 = 0$

$\Rightarrow 16x^2 = -25$

$\Rightarrow x^2 = -\frac{25}{16}$

No solutions

$(x+1)^2 = -5$

↓

No solutions

You will not be testing over imaginary numbers.

$\boxed{\text{Quadratic Formula}}$

$ax^2 + bx + c = 0$ [General form of a quadratic eqn.]

a, b, c are numbers.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-b + \sqrt{b^2-4ac}}{2a} \text{ or } x = \frac{-b - \sqrt{b^2-4ac}}{2a}$$

"Solve by completing the squares" $\Rightarrow$ use quadratic formula.

Ex.

1. Let $g(x) = 2x^2 + x$ find a so that $g(a) = 28$

$\Rightarrow g(a)=28 \Rightarrow 2a^2 + a = 28 \Rightarrow \underbrace{2}_{}a^2 + \underset{1}{a} \underbrace{-28}_{-28} = 0$

$\Rightarrow a = \dfrac{-1 \pm \sqrt{(-1)^2 - 4(2)(-28)}}{2(2)} = \dfrac{-1 \pm \sqrt{1+224}}{4} = \dfrac{-1 \pm \sqrt{225}}{4} = \dfrac{-1 \pm 15}{4} \Rightarrow a = \dfrac{-1+15}{4}$ or $\dfrac{-1-15}{4}$

$\Rightarrow a = \dfrac{7}{2}, -4$

2. Let $g(x) = 15x + x^2$ find a so that $g(a) = -56$

$a^2 + 15a = -56 \Rightarrow a^2 + 15a + 56 = 0$

$\Rightarrow a = -7, -8$

3. Let $f(x) = x^2 - x$ find a so that $g(x) = 36 + 4x$ find all the x-values for which $f(x) = g(x)$

$x^2 - x = 36 + 4x \Rightarrow x^2 - x - 36 - 4x = 0$

$\Rightarrow \underbrace{x^2}_{a=1} \underbrace{-5x}_{b=-5} \underbrace{-36}_{c=-36} = 0 \Rightarrow x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-36)}}{2} = \dfrac{5 \pm \sqrt{25+144}}{2} = \dfrac{5 \pm \sqrt{169}}{2} = \dfrac{5 \pm 13}{2}$

$x = 9$ or -4

Problem Solving

ESSENTIALS

The Compound-Interest Formula: If an amount of money P is invested at interest rate r, compounded annually, in t years it will grow to the amount A given by $A = P(1+r)^t$.

Distance of a Free Fall: $s = 16t^2$ approximates the distance s, in feet, that an object falls freely from rest in t seconds.

INTEREST

Grady invested \$5000 at interest rate r, compounded annually. In 2 years, it grew to \$5304.50. Find the interest rate.

$P = 5000, \; t = 2, \; A = 5304.50$

$\Rightarrow 5304.50 = 5000(1+r)^2 \Rightarrow (r+1)^2 = \dfrac{5304.50}{5000} = 1.0609$

$\Rightarrow r + 1 = \pm\sqrt{1.0609} = \pm 1.03 \Rightarrow r = 1.03 - 1$ or $\cancel{-1.03 - 1}$ $(r > 0)$ $\Rightarrow r = 0.03$

Betsy invested \$10,000 at interest rate r, compounded annually. In 2 years, it grew to \$11,236. Find the interest rate.

$s = 16t^2$

$\nearrow s \qquad \nearrow t = ?$

Brenna dropped a stone from a bridge 2000 ft above a river. How long did it take the stone to reach the river? Round to the nearest tenth of a second. $\Rightarrow 16t^2 = 2000 \quad \Rightarrow t^2 = \frac{2000}{16} \quad \Rightarrow t^2 = 125$ $\Rightarrow t = \pm\sqrt{125} \quad \Rightarrow t = 11.18$ seconds   time cannot be -ve
Gav dropped a penny from a bridge 500 ft above a river. How long did it take the penny to reach the river? Round to the nearest tenth of a second. $s = 500$ $16t^2 = 500$