- Build a quad function
- $x = -b/2a$

Math-I 110 8.8 Maximum and Minimum Problems

1. Newborn Calves

The number of pounds of milk per day recommended for a calf that is x weeks old can be approximated by $p(x) = -0.2x^2 + 1.3x + 6.2$.

$a = -0.2,\ b = 1.3,\ c = 6.2$

When is a calf's milk consumption greatest and how much milk does it consume at that time?

$a = -0.2 < 0 \Rightarrow P(x)$ will have a max value

$h = \frac{-b}{2a}, \quad \boxed{k = \frac{4ac - b^2}{4a}}$ Alternatively $= \frac{4(-0.2)(6.2) - (1.3)^2}{4(-0.2)} = 8.3$ Pounds

$h = \frac{-1.3}{2(-0.2)} = \frac{1.3}{0.4} = 3.25$ weeks.

$\Rightarrow P(3.25) = -0.2(3.25)^2 + 1.3(3.25) + 6.2$

$= 8.3$ Pounds.

How to:

- Identify coefficients a = -0.2, b = 1.3.
- Find the vertex using x = -b / (2a).
- Substitute x into p(x) to find the maximum milk consumption.
- State both x (weeks) and p(x) (pounds).

2. Stock Prices

The value of a share of I. J. Solar can be represented by $V(x) = x^2 - 6x + 13$, where x is the number of months after January 2023.

$a = 1,\ b = -6,\ c = 13$

What is the lowest value V(x) will reach, and when did that occur?

$a > 0$ ✓ $\Rightarrow$ will reach a min. value.

$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3 \leftarrow$ the min. occurs at 3 months.

$\Rightarrow V(3) = 3^2 - 6(3) + 13 = 4$

min value

How to:

- Identify a = 1, b = -6.
- Since a > 0, the vertex gives the minimum.
- Use x = -b / (2a) and substitute to find V(x).
- Interpret x as months after January 2023.

3. Minimizing Cost

Sweet Harmony Crafts determined that when x hundred dulcimers are built, the average cost per dulcimer is $C(x) = 0.1x^2 - 0.7x + 2.425$ (in hundreds of dollars).

Find the $\underbrace{\text{minimum average cost per}}_{\text{min value}}$ dulcimer and $\underbrace{\text{how many dulcimers should be built}}_{x \text{ at which min occurs}}$.

$C(x) = 0.1x^2 - 0.7x + 2.425 \Rightarrow a = 0.1,\ b = -0.7,\ c = 2.425$

$a > 0 \Rightarrow$ will have a min. At $x = \frac{-(-0.7)}{2(0.1)} = \frac{0.7}{0.2} = 3.5$

$\Rightarrow$ min avg cost $= C(3.5) = 0.1(3.5)^2 - 0.7(3.5) + 2.425$

$= 1.2$

$\Rightarrow$ 350 dulcimers should be built
avg cost would be \$ 120.

How to:
- Identify a = 0.1, b = -0.7.
- Find vertex x = -b / (2a).xa
- Substitute into C(x) to find the minimum cost.

4. Maximizing Profit

Given $R(x) = 1000x - x^2$ and $C(x) = 3000 + 20x$, find the total profit $P(x) = R(x) - C(x)$. Find the maximum profit and the value of x at which it occurs.

$P(x) = R(x) - C(x) = 1000x - x^2 - 3000 - 20x$

$\boxed{a = -1,\ b = 980,\ c = -3000}$ $= -x^2 + 980x - 3000$

$a < 0 \Rightarrow$ will have a max at $x = \frac{-980}{2(-1)} = \boxed{490}$ units need to sold

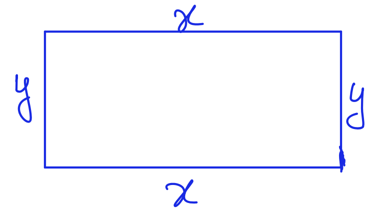
$P(490) = -(490)^2 + 980(490) - 3000 = \boxed{237100}$

max profit

How to:
- Write P(x) = R(x)-C(x)
- Simplify to get a quadratic.
- Find the vertex to determine maximum profit.

5. Architecture

A hotel atrium is rectangular with a perimeter of 720 ft of brass piping. What dimensions maximize the area?



$2x+2y=720 \Rightarrow 2(x+y)=720$

$\Rightarrow x+y=360$

$\Rightarrow y=360-x$

$A=xy$

$A(x)=x(360-x)$

$=360x-x^2=\underbrace{-x^2+360x}_{\text{want to maximize}}$

$\Rightarrow a=-1,\ b=360,\ c=0$

$a<0 \Rightarrow$ will have a max

max at $x=\frac{-360}{2(-1)}$

$x=180$

$y=360-180$

$=180$

$\Rightarrow$ Dimensions that maximize area are $x=y=180$

How to:

- Let width = w and length = l.
- Find perimeter equation
- Express area as A = lw in one variable.
- Find vertex to get maximum area.

6. Furniture Design

A furniture builder is designing a rectangular end table with a perimeter of 128 in. What dimensions yield the maximum area?

$2x+2y=128 \Rightarrow 4x=128$

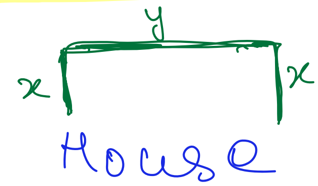
Max. area at $x=y=32$ inches.

How to:

- Same process as the architecture problem.
- Use perimeter formula and maximize A = lw.

7. Patio Design

A mason can build a rectangular patio with a 60 ft perimeter, assuming the house forms one side. What is the maximum area and the dimensions?



$\Rightarrow 2x + y = 60 \Rightarrow y = 60 - 2x$

$A = xy = x(60-2x)$

$\Rightarrow A(x) = -2x^2 + 60x$

$a = -2,\ b = 60,\ c = 0$

$a < 0 \Rightarrow$ will have max at $x = \frac{-60}{2(-2)} = 15$ ft.

$y = 60 - 2(15) = 30$ ft.

How to:

- Only 3 sides use fencing.
- Write perimeter
- Substitute into A = lw and maximize.

Dimensions maximizing area are $x = 15,\ y = 30$

Maximum area $= 15(30) = 450$ sq.ft.

8. Garden Design

Tallie is fencing a rectangular garden using the house as one side with 40 ft of fencing. What dimensions yield the maximum area?

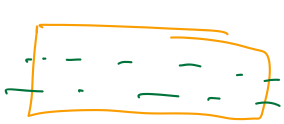
Perimeter

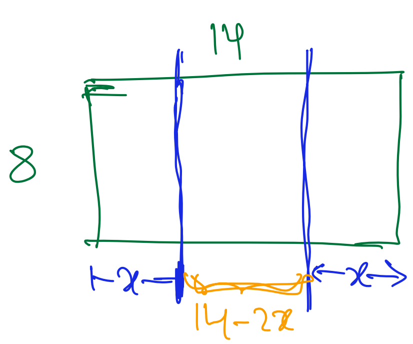
HW.

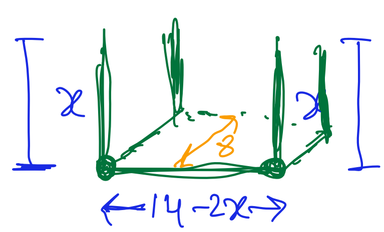
ans. $x = 10,\ y = 20$

9. Molding Plastics

Economite Plastics bends an 8-by-14-inch sheet along two lines to form a U-shape. How tall should the sides be to maximize volume?







Cuboid of shape $\underset{h}{x}$ by $\underset{l}{14-2x}$ by $\underset{w}{8}$

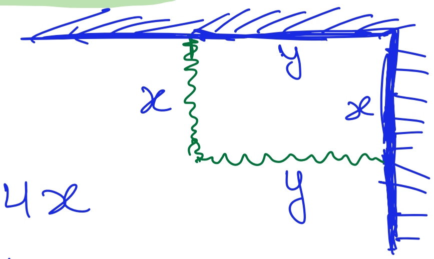
$V(x) = (14-2x)8x = -16x^2 + \overbrace{112}^{b}x$

$a = -16 < 0 \Rightarrow$ max. at $x = -112/2(-16) = 3.5$ inch.

$\Rightarrow$ sides should be 3.5 inches tall to maximize volume

10. Composting

A rectangular compost container uses 8 ft of chicken wire for two sides, and the other sides meet a corner. The chicken wire is 3 ft high. What base dimensions maximize volume?



$x + y = 8 \Rightarrow y = 8 - x$

$V(x) = x(8-x)3 = -3x^2 + 24x$

$a = -3 < 0 \Rightarrow$ max at $x = \frac{-24}{2(-3)} = 4$ ft.

$\Rightarrow$ Base dimensions maximizing volume are $x = y = 4$ ft.

11. Two Numbers Add to 18

What is the maximum product of two numbers that add to 18?

$x + y = 18 \Rightarrow y = 18 - x$

$P(x) = x(18-x) \Rightarrow P(x) = -x^2 + 18x \Rightarrow a = -1, b = 18$

$a < 0 \Rightarrow$ max at $x = \frac{-18}{2(-1)} = 9 \Rightarrow y = 18 - 9 = 9$

$\Rightarrow$ Both numbers should be 9

$\Rightarrow$ max. product $= 9^2 = 81$

12. Two Numbers Add to 26

What is the maximum product of two numbers that add to 26?

ans. $\rightarrow 13^2 = 169$

13. Two Numbers Differ by 8

What is the minimum product of two numbers that differ by 8?

$x - y = 8 \Rightarrow x = y + 8$

$P(y) = xy = (y+8)y \Rightarrow P(y) = y^2 + 8y$

$a = 1, b = 8, c = 0 \Rightarrow a > 0 \Rightarrow$ min. occurs at $y = \frac{-8}{2(1)}$

$\Rightarrow y = -4$

$\Rightarrow x = -4 + 8 = 4 \Rightarrow$ min. Product $= 4(-4) = -16$