Math 110- 9.2 Notes

$f(x) = x^a$ — a fixed number; $x^2, x^3, x^{5/2}$; input

Graphing Exponential Functions

ESSENTIALS

The function $f(x) = a^x$, where a is a positive constant different from 1, is called an **exponential function**, base a. $a \neq 1, a > 0$

All functions $f(x) = a^x$ go through the point $(0,1)$. That is, the y-intercept is $(0,1)$.

$a^0 = 1$

$a = 1$: $f(x) = 1^x = 1$

a^x, a → fixed number

$0 < a < 1$: decreasing function

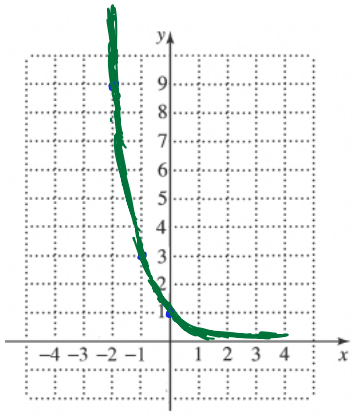
$a > 1$: increasing function

Graph the following functions using a table

$f(x) = \left(\frac{1}{3}\right)^x$

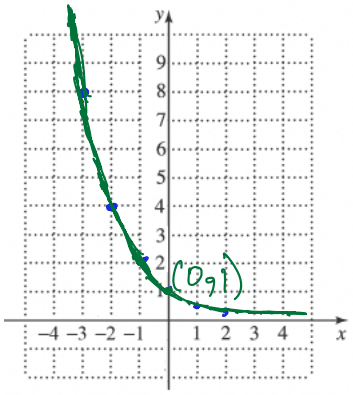
x	$f(x)$
0	1
−1	3
−2	9

$\left(\frac{1}{3}\right)^{-2} = 3^2 = 9$



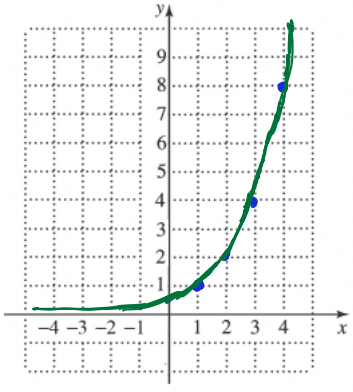
$f(x) = \left(\frac{1}{2}\right)^x$

x	$f(x)$
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
−1	$\left(\frac{1}{2}\right)^{-1} = 2$
−2	$\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$
−3	$\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$



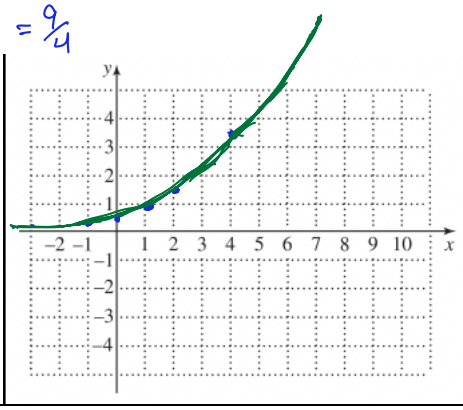
$f(x) = (2)^{x-1}$

x	$f(x)$
1	$2^{1-1} = 2^0 = 1$
2	$2^{2-1} = 2$
3	$2^{3-1} = 4$
4	$2^{4-1} = 8$
0	$2^{0-1} = \frac{1}{2}$
−1	$2^{-1-1} = \frac{1}{4}$



$x = \left(\frac{3}{2}\right)^{x-1}$ → $a = 1.5$

x	$f(x)$
1	$\left(\frac{3}{2}\right)^{1-1} = \left(\frac{3}{2}\right)^0 = 1$
2	$\left(\frac{3}{2}\right)^{2-1} = \frac{3}{2}$
3	$\left(\frac{3}{2}\right)^{3-1} = \frac{9}{4}$
4	$\left(\frac{3}{2}\right)^{4-1} = \frac{27}{8}$
0	$\left(\frac{3}{2}\right)^{0-1} = \frac{2}{3}$
−1	$\left(\frac{3}{2}\right)^{-1-1} = \frac{4}{9} \approx 0.44$



$5\% \Rightarrow r = \frac{5}{100} = 0.05$

Applications of Exponential Functions

The amount of money A that a principal P will be worth after t years at an interest rate r, compounded annually, is given by $A = P(1+r)^t$. Suppose that $5000 is invested at 5% interest, compounded annually.

↳ t in years.

a. Find a function for the amount in the account after t years

$$A(t) = 5000\left(1+\frac{5}{100}\right)^t = 5000(1.05)^t$$

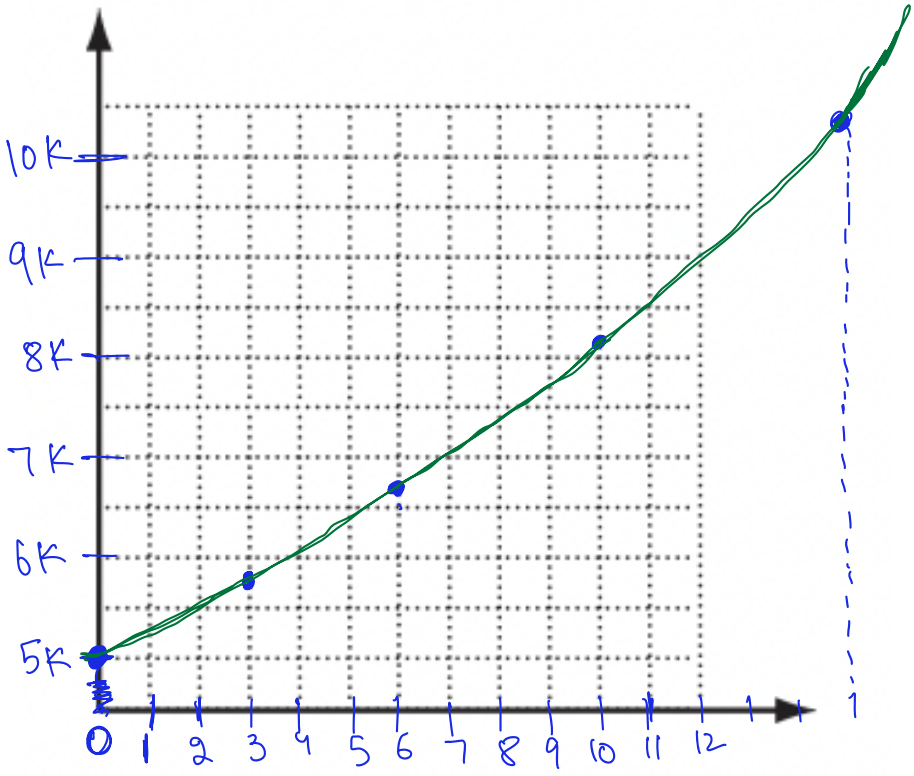
b. Find the amount of money in the account at t=0, t=3, t=6, and t=10

t	A(t)
0	$5000(1.05)^0 = 5000$
3	$5000(1.05)^3 = 5788$
6	$5000(1.05)^6 = 6700$
10	$5000(1.05)^{10} = 8144$
15	$5000(1.05)^{15} = 10394$

$r = 10\%$

A doubles in every 7 yrs.

c. Graph the function



$2^x \rightarrow (1+1)^x$

$r = 1$

$r = 100\%$