9.1 $\Rightarrow$ Inverse functions

9.2 $\Rightarrow$ Exponential function.

Math 110 – 9.3 Notes

Meaning Of A Logarithm

Consider the exponential function $f(x) = 2^x$. Like all exponential functions f is one-to-one. Let's attempt to find a formula for $f^{-1}(x)$:

1. Replace $f(x)$ with y
2. Interchange x and y
3. Solve for Y
4. Replace y with $f^{-1}(x)$

$y = f(x) \Rightarrow y = 2^x$

$\boxed{x = 2^y}$

y= the exponent to which we raise 2 to get x

$x \to$ input

$y \to$ output

Want to find y for a given x

One-to-one functions always have an inverse.

We now define a new symbol to replace the words "the exponent to which we raise 2 to get x"

$\log_2 x$ is read ' the logarithm, base 2, of x" or " log, base 2" means "the exponent to which we raise 2 to get x"

Simplifying Logarithms Inverse to $f(x) = a^x$ $(a > 0, a \neq 1)$ is $f^{-1}(x) = \log_a x$

Exponential functions are One-to one

1. Simplify $\log_2 32$ by using the definition of a logarithm.

$y =$ $\log_2 32$ means : the exponent to which we raise __2__ to get __32__ : y such that $2^y = 32$

Therefore, $\log_2 32 =$ __5__

2. Simplify $\log_{10} 1000$ by using the definition of a logarithm.

$\log_{10} 1000$ is y for which $10^y = 1000 = 10^3$

$\Rightarrow y = 10 \Rightarrow \log_{10} 1000 = 3$

3. Simplify $\log_7 49$ by using the definition of a logarithm.

$\log_7 49$ is y for which $7^y = 49 = 7^2$

$\Rightarrow y = 2 \Rightarrow \log_7 49 = 2$

4. Simplify $\log_8 \frac{1}{8}$ by using the definition of a logarithm.

$\log_8 \frac{1}{8}$ is y for which $8^y = \frac{1}{8} = 8^{-1}$

$\Rightarrow y = -1 \Rightarrow \log_8 \left(\frac{1}{8}\right) = -1$

5. Simplify $\log_{16} 4$ by using the definition of a logarithm.

$\log_{16} 4$ is y for which $16^y = 4 = 16^{1/2}$

$\Rightarrow y = \frac{1}{2} \Rightarrow \log_{16} 4 = \frac{1}{2}$

6. Simplify $\log_6 6^5$ by using the definition of a logarithm.

$\log_6 6^5$ is y for which $6^y = 6^5$

$\Rightarrow y = 5$

$\Rightarrow \log_6 6^5 = 5$

$x = 0 \Rightarrow 2^y = 0$

$\Rightarrow$ not Possible

$x = 1 \Rightarrow 2^y = 1 = 2^0$

$\Rightarrow y = 0$

$x = 2 \Rightarrow 2^y = 2 = 2^1$

$\Rightarrow y = 1$

$x = 4 \Rightarrow 2^y = 4 = 2^2$

$\Rightarrow y = 2$

y for which $2^y = x$

$x = 1$ $\boxed{\log_2}$ $y = 0$

$x = 2$ $\boxed{\log_2}$ $y = 1$

$x = 4$ $\boxed{\log_2}$ $y = 2$

$\vdots$

$x = 1/2$ $\boxed{\log_2}$ $y = -1$

$x = 1/4$ $\boxed{\log_2}$ $y = -2$

Cancellation eqns.: $f(f^{-1}(x))=x$ and $f^{-1}(f(x))=x$

$a^{\log_a x}=x$ $\log_a(a^x)=x$

Example: How would we simplify $3^{\log_3 29}$?

$3^{\log_3 29}=29$

Definition of a Logarithmic Function

For x > 0 and b > 0, b≠ 1,

$$y = \log_b x \text{ is equivalent to } b^y = x$$

The function $f(x) = \log_b x$ is the **logarithmic function with base b.**

Logarithmic Form	Exponential Form
$y = \log_b x$ (Exponent: y; Base: b)	$b^y = x$. (Exponent: y; Base: b)

$5=\log_2 32$ / $2^5=32$

Note: that logarithm bases must be positive because logarithms are defined as using exponential functions that require positive bases.

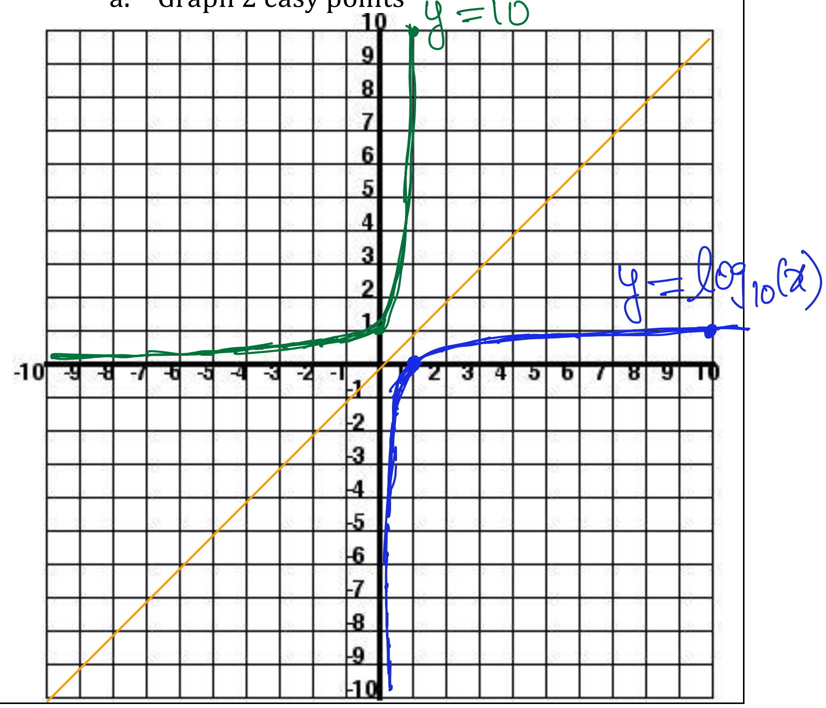
Graphs of Logarithmic Functions

1. Graph $y = \log_{10} x$

a. Create a table using the following x values, then use the rules we learned about simplifying to get the y values

X	Y $=\log_{10}(x)$
10^{-2} = 0.01	−2
10^{-1} = 0.1	−1
10^{0} = 1	0
10^{1} = 10	1
10^{2} = 100	2

a. Graph 2 easy points

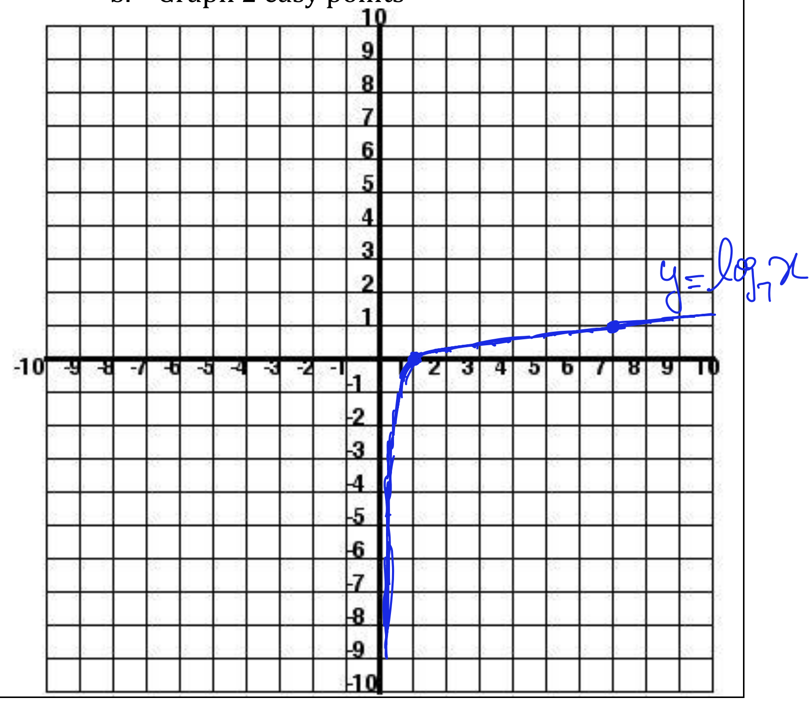


2. Graph $y = \log_7 x$

b. Create a table using the following x values, then use the rules we learned about simplifying to get the y values

X	Y
$7^{-2} = 1/49$	-2
$7^{-1} = 1/7$	-1
$7^{0} = 1$	0
$7^{1} = 7$	1
$7^{2} = 49$	2

b. Graph 2 easy points

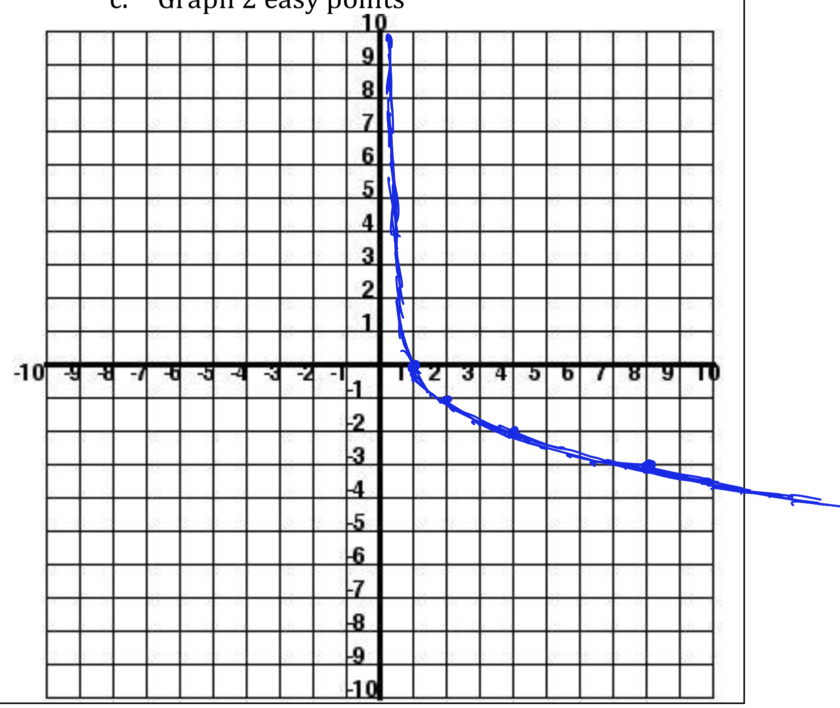


3. Graph $y = \log_{\frac{1}{2}} x$

c. Create a table using the following x values, then use the rules we learned about simplifying to get the y values

X	Y
$(1/2)^{-2} = 4$	-2
$(1/2)^{-1} = 2$	-1
$(1/2)^{0} = 1$	0
$(1/2)^{1} = 1/2$	1
$(1/2)^{2} = 1/4$	2

c. Graph 2 easy points



Graphing Exponential and Logarithmic Functions

Graph $f(x) = 3^x$ and $f(x) = \log_3 x$ on the same axis

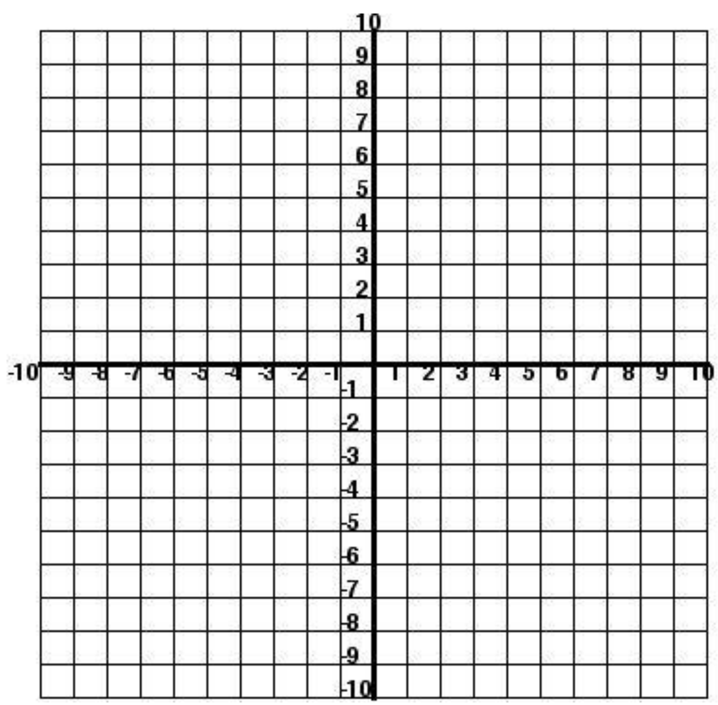
HW: (same as in $\log_{10} x$)

Table for $f(x) = 3^x$

X	Y
-2	
-1	
0	
1	
2	

Table for $f^{-1}(x) = \log_3 x$

X	Y
$\_\_\_^{-2} = \_\_\_\_$	
$\_\_\_^{-1} = \_\_\_\_$	
$\_\_\_^{0} = \_\_\_\_$	
$\_\_\_^{1} = \_\_\_\_$	
$\_\_\_^{2} = \_\_\_\_$	



$y = \log_a x \iff a^y = x$

Finding Equivalent Expressions of a Logarithm

Use the definition of a logarithm to rewrite a logarithmic equation as an equivalent exponential equation or the other way around.

$$\log_b(a) = c \iff b^c = a$$

1. $\log_{10} 8 = y$

$a = 8,\ b = 10,\ c = y$

$10^y = 8$

2. $\log_9 9 = 1$

$\Rightarrow 9^1 = 9$

3. $10^x = 100$

$\Rightarrow \log_{10} 100 = x$

4. $z^m = 6$

$\Rightarrow \log_z 6 = m$

Solving Logarithmic Equations

1. Rewrite the logarithm in its exponential form
2. Use the following property to solve $b^m = b^n$ is equivalent to $m = n$

Solve.

1. $\log_6 x = 2$

$\Rightarrow x = 6^2 \Rightarrow x = 36$

2. $\log_4 x = 3$

$\Rightarrow x = 4^3 \Rightarrow x = 64$

3. $\log_x 9 = 1$

$\Rightarrow 9 = x^1$

$\Rightarrow x = 9$

4. $\log_x 7 = \frac{1}{2}$

$\Rightarrow 7 = x^{1/2}$

$\Rightarrow x^{1/2} = 7$

$\Rightarrow (x^{1/2})^2 = 7^2 \Rightarrow x = 49$

5. $\log_2 32 = x$

$\Rightarrow 32 = 2^x$

$\Rightarrow 2^x = 32 = 2^5$

$\Rightarrow x = 5$

6. $\log_5 25 = x$

$\Rightarrow 25 = 5^x$

$\Rightarrow 5^x = 25 = 5^2$

$\Rightarrow x = 2$