$\log_3 4 + \log_2 5 \neq \log_{11} 20$

Math 11000 – 9.4 Notes

9.4 Properties of Logarithmic Functions

$\log_2 4 + \log_2 3 = \log_2 12$

Logarithmic of Products

$\log_3 15 = \log_3 5 + \log_3 3$

a. Express as a single logarithm

PRODUCT Property $\log_b(m \cdot n) =$ $= \log_b m + \log_b n$	**1.** $\log_3 9 + \log_3 5$ $= \log_3(9 \cdot 5)$ $= \log_3 45$	**2.** $\log 6 + \log(x-3)$ $= \log 6(x-3)$ $= \log(6x-18)$ ~~$\log 6x - 18$~~	**3.** $\log_t H + \log_t M$ $= \log_t (HM)$ $= \log_t HM$

b. Express the single logarithms as a sum

1. $\log_c(rst)$	2. $\log_c(3ab)$	3. $\log_x(10x)$	4. $\log_2(35)$
$= \log_c r + \log_c(st)$ $= \log_c r + \log_c s + \log_c t$	$= \log_c 3 + \log_c(ab)$ $= \log_c 3 + \log_c a + \log_c b$	$= \log_x 10 + \log_x x$ $= \log_x(2 \cdot 5) + \log_x x$ $= \log_x 2 + \log_x 5 + \log_x x$	$= \log_2(5 \cdot 7)$ $= \log_2 5 + \log_2 7$

c. Express as a product

POWER Property $\log_b m^n =$ $= n \log_b m$	1. $\log_a r^8$ $= 8 \log_a r$	2. $\log_2 y^{\frac{1}{3}}$ $= \frac{1}{3} \log_2 y$	3. $\log_2 x^3 y^6$ $= \log_2 (xy^2)^3$ $= 3 \log_2 (xy^2)$

d. Express as a power

5. $3 \log x$	6. $\frac{1}{2}\log_4 81$	7. $-5 \log_c M$	8. $-3 \log_4 2$
$= \log x^3$	$= \log_4 81^{\frac{1}{2}}$ $= \log_4 9$	$= \log_c M^{-5}$	$= \log_4 2^{-3}$ $= \log_4 \left(\frac{1}{8}\right)$

e. Express as a difference of logarithms

QUOTIENT Property $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$	9. $\log_2\left(\frac{5}{11}\right)$ $= \log_2 5 - \log_2 11$	10. $\log_2\left(\frac{29}{13}\right)$ $= \log_2 29 - \log_2 13$	11. $\log_b\left(\frac{m}{n}\right)$ $= \log_b m - \log_b n$

f. Express as an equivalent expression, using the individual logarithms

$\log_a(wxy)$

$= \log_a w + \log_a x + \log_a y$

$\log_a(x^2y^5)$

$= \log_a x^2 + \log_a y^5$

$= 2\log_a x + 5\log_a y$

$\log_a(xy^2z^{-3})$

$= \log_a x + \log_a y^2 + \log_a z^{-3}$

$= \log_a x + 2\log_a y - 3\log_a z$

$\log_a\left(\frac{x^4}{yz^2}\right)$

$= \log_a x^4 - \log_a yz^2$

$= 4\log_a x - (\log_a y + \log_a z^2)$

$= 4\log_a x - \log_a y - \log_a z^2$

$= 4\log_a x - \log_a y - 2\log_a z$

$\log_b\left(\frac{w^2x}{y^3z}\right)$

$= \log_b w^2x - \log_b y^3z$

$= \log_b w^2 + \log_b x - \log_b y^3 - \log_b z$

$= 2\log_b w + \log_b x - 3\log_b y - \log_b z$

$\log_c\left(\sqrt{\frac{x^4}{y^3z^2}}\right)$

$= \log_c\left(\frac{x^4}{y^3z^2}\right)^{1/2} = \frac{1}{2}\left(\log_c \frac{x^4}{y^3z^2}\right)$

$= \frac{1}{2}\left(\log_c x^4 - \log_c y^3z^2\right)$

$= \frac{1}{2}\left(4\log_c x - \log_c y^3 - \log_c z^2\right)$

$= \frac{1}{2}\left(4\log_c x - 3\log_c y - 2\log_c z\right)$

$= 2\log_c x - \frac{3}{2}\log_c y - \log_c z$

$\log_a\left(\sqrt[4]{\frac{x^8y^{12}}{a^3z^5}}\right)$

$= \log_a\left(\frac{x^8y^{12}}{a^3z^5}\right)^{1/4} = \frac{1}{4}\left(\log_a \frac{x^8y^{12}}{a^3z^5}\right)$

$= \frac{1}{4}\left(\log_a x^8y^{12} - \log_a a^3z^5\right)$

$= \frac{1}{4}\left(\log_a x^8 + \log_a y^{12} - \log_a a^3 - \log_a z^5\right)$

$= \frac{1}{4}\left(8\log_a x + 12\log_a y - 3\log_a a - 5\log_a z\right)$

$= 2\log_a x + 3\log_a y - \frac{3}{4}\log_a a - \frac{5}{4}\log_a z$

$\log_b b = 1$ $\log_b 1 = 0$

g. Express as a single logarithm and if possible, simplify

$\log_a x + \log_a 3$

$= \log_a(3x)$

$\log_a c - \log_a d$

$= \log_a\left(\frac{c}{d}\right)$

$8\log_a x + 3\log_a z$

$= \log_a x^8 + \log_a z^3$

$= \log_a(x^8 z^3)$

$2\log w - \log z - 4\log y$

$= \log w^2 - \log z - \log y^4$

$= \log w^2 - (\log z + \log y^4)$

$= \log w^2 - \log(zy^4)$

$= \log\left(\frac{w^2}{zy^4}\right)$

$3\log p + \frac{1}{2}\log t - \log 7$

$= \log p^3 + \log t^{1/2} - \log 7$

$= \log(p^3 t^{1/2}) - \log 7$

$= \log\left(\frac{p^3 t^{1/2}}{7}\right) = \log\left(\frac{p^3\sqrt{t}}{7}\right)$

$\log_a(x^2 - 25) - \log_a(x + 5)$

$= \log_a\left(\frac{x^2-25}{x+5}\right)$

$= \log_a\left(\frac{(x-5)(x+5)}{(x+5)}\right)$

$= \log_a(x-5)$

$\log_a(x^2 - 36) - \log_a(2x + 12)$

$= \log_a \frac{x^2-36}{2x+12}$

$= \log_a \frac{(x-6)(x+6)}{2(x+6)}$

$= \log_a\left(\frac{x-6}{2}\right)$