

Equations with variable in exponents such as $5^x = 12$ and $2^{7x} = 64$, are called exponential equations. We can solve certain exponential equations by using the principle of exponential equality.

Solving Exponential Equations With Same Bases



$$a^x = a^y \iff x = y$$

Example:

$$5^x = 5^3 \iff x = 3$$

Example: Solve $4^{3x} = 16$

$$4^{3x} = 4^2$$

$$\Rightarrow 3x = 2$$

$$\Rightarrow x = \frac{2}{3}$$

Example: Solve $3^{4x} = 9$

$$\Rightarrow 3^{4x} = 3^2$$

$$\Rightarrow 4x = 2$$

$$\Rightarrow x = \frac{2}{4} \Rightarrow x = \frac{1}{2}$$

If we cannot write both sides of equation as powers of the base, we will need to use logarithms to solve the equations. (you may use Log or Ln in order to solve this).

Example: Solve $7^{x-2} = 60$

• Take log on both sides.

$$\log 7^{x-2} = \log 60$$

$$\Rightarrow (x-2) \log 7 = \log 60$$

$$\Rightarrow x-2 = \frac{\log 60}{\log 7}$$

$$\Rightarrow x = 2 + \frac{\log 60}{\log 7} = 4.10$$

Example: Solve $5^{x+1} = 12$

$$\Rightarrow \log 5^{x+1} = \log 12$$

$$\Rightarrow (x+1) \log 5 = \log 12$$

$$\Rightarrow x+1 = \frac{\log 12}{\log 5}$$

$$\Rightarrow x = -1 + \frac{\log 12}{\log 5}$$

$$\Rightarrow x = 0.54$$

Solving with base e

$$\boxed{\ln e = 1} \Rightarrow \ln e = \log_e e = 1$$

Example: Solve $e^{0.06t} = 1500$

• Take $\ln$ on both sides.

$$\ln e^{0.06t} = \ln(1500)$$

$$\Rightarrow 0.06t \boxed{\ln e} = \ln 1500$$

$= 1$

$$\Rightarrow 0.06t = \ln 1500$$

$$\Rightarrow t = \frac{\ln 1500}{0.06}$$

$$\Rightarrow t = 121.89$$

Example: Solve $e^{-0.03t} = 120$

$$\Rightarrow \ln e^{-0.03t} = \ln 120$$

$$\Rightarrow -0.03t \boxed{\ln e} = \ln 120$$

$= 1$

$$\Rightarrow -0.03t = \ln 120$$

$$\Rightarrow t = \frac{\ln 120}{-0.03}$$

$$\Rightarrow t = -159.58$$