Math 110 Notes **9.7**

rate $= r$ % per annum then after t years.

$$\boxed{A(t) = P_0\left(1+\frac{r}{100}\right)^t}$$

Doubling Time

$$P(t) = P_0 e^{kt}$$

1. Suppose that $60,000 is invested at 5% interest, compounded annually. After time t, in years, it ground to the amount A given by the function $A(t) = 60{,}000(1.05)^t$

Find the amount of time after which there will be $480,000 in the amount.

t when $A(t) = 480\,000 \Rightarrow 60\,000(1.05)^t = 480\,000$

$\Rightarrow (1.05)^t = \frac{\cancel{480000}^{8}}{\cancel{60000}} \Rightarrow (1.05)^t = 8 \Rightarrow \log 1.05^t = \log 8$

$\Rightarrow t \log 1.05 = \log 8 \Rightarrow t = \frac{\log 8}{\log 1.05} \Rightarrow t = 42.62$ yrs.

Find the doubling time

t when $A(t) = 2(60000)$

$\Rightarrow 60\,000(1.05)^t = 2(60000) \Rightarrow 1.05^t = 2 \Rightarrow t \log 1.05 = \log 2 \Rightarrow t = \frac{\log 2}{\log 1.05} \Rightarrow t = 14.21$ yrs.

2. Suppose that P is invested in a savings account in which interest, k, is compounded continuously 5% per year. The balance P(t) after time t, in years, is $P(t) = P_0 e^{kt}$

$k \longrightarrow$ rate of interest.

$\Rightarrow k = \frac{5}{100} = 0.05$

What is ~~that~~ the exponential growth function in terms of P_0 and 0.05?

$\Rightarrow P(t) = P_0 e^{0.05t}$

If $8000 is invested what is the balance after 7 years

$P(7) = 8000\, e^{0.05(7)} = 8000\, e^{0.35}$

$= 11352.54$

How many years does it take to double the $8000

t for which $P(t) = 2(8000)$

$\Rightarrow 8000\, e^{0.05t} = 2(8000) \Rightarrow e^{0.05t} = \frac{2(8000)}{8000} = 2$

$\Rightarrow e^{0.05t} = 2 \Rightarrow \ln e^{0.05t} = \ln 2$

$\Rightarrow 0.05t \underbrace{\boxed{\ln e}}_{=1} = \ln 2 \Rightarrow t = \frac{\ln 2}{0.05} \Rightarrow t = 13.86$ yrs.

3. The population of a city was 177,000 in 1992. The exponential growth rate was 1.3% per year.

$P_0 = 177000 \quad , \quad k = \frac{1.3}{100} \Rightarrow k = 0.013$

Find the exponential growth function in terms of t , where t is the number of years since 1992

$\Rightarrow P(t) = 177000\, e^{0.013t}$

Predict the population in 2012, to the nearest thousand

$t = 2012 - 1992 = 20$

$\Rightarrow P(20) = 177000\, e^{0.013(20)} = 177000\, e^{0.26} = 229556.6$

≈ 229557

In what year will the population be 194 thousand

t for which $P(t) = 194000 \Rightarrow 177000\, e^{0.013t} = 19400$

$\Rightarrow e^{0.013t} = \frac{194000}{177000} = \frac{194}{177} \Rightarrow \ln e^{0.013t} = \ln \frac{194}{177} \Rightarrow 0.013t = \ln \frac{194}{177}$

$\Rightarrow t = \left(\ln \frac{194}{177}\right) / 0.013 = 7.05 \approx 7$ yrs. $\Rightarrow$ yr $= 1992 + 7 = 1999$

4. In 2016, the exponential growth rate of the population of a city was 3.56% per year. What was the doubling time?

$k = \frac{3.56}{100} = 0.0356$

$P(t) = P_0\, e^{0.0356t}$

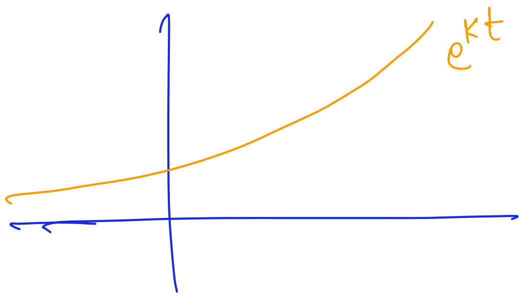
t for which $P(t) = 2P_0 \Rightarrow \cancel{P_0}\, e^{0.0356t} = 2\cancel{P_0}$

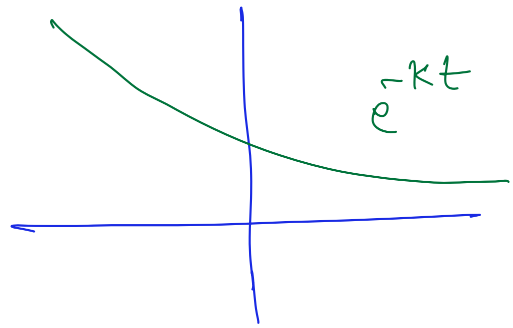
$\Rightarrow e^{0.0356t} = 2 \Rightarrow \ln e^{0.0356t} = \ln 2$

$\Rightarrow 0.0356t\, \underbrace{\boxed{\ln e}}_{=1} = \ln 2 \Rightarrow t = \frac{\ln 2}{0.0356}$

$\Rightarrow t = 19.47$ yrs. $\Rightarrow$ yr $\rightarrow 2016 + 19.47 = 2035.47$

≈ 2035





5. The amount of carbon 14 present in animal bones after t years is given by $P(t) = P_0e^{-0.00012t}$. A bone has lost 32% of its carbon 14. How old is the bone?

Initially amount was P_0.

Amount Lost $= \frac{32}{100} P_0 = 0.32 P_0 \Rightarrow$ Amount left $= P_0 - 0.32P_0 = 0.68 P_0$

t for which $P(t) = 0.68 P_0 \Rightarrow \cancel{P_0} e^{-0.00012t} = 0.68 \cancel{P_0}$

$\Rightarrow e^{-0.00012t} = 0.68 \Rightarrow \ln e^{-0.00012t} = \ln 0.68 \Rightarrow -0.00012t = \ln 0.68$

$\Rightarrow t = \frac{\ln 0.68}{-0.00012} \Rightarrow t = 3213.85 \simeq$ 3214 years.

6. The decay rate of a certain chemical is 9.3% per year. What is its half-life? Use the exponential decay model $P(t) = P_0e^{-kt}$ where k is the decay rate, and P_0 is th original amount of chemical

Half-life is t after which $P(t) = \frac{1}{2}P_0$, $k = \frac{9.3}{100} = 0.093$

$\Rightarrow$ t for which $P(t) = \frac{1}{2}P_0 \Rightarrow \cancel{P_0} e^{-0.093t} = \frac{1}{2}P_0 = 0.5\cancel{P_0}$

$\Rightarrow e^{-0.093t} = 0.5 \Rightarrow \ln(e^{-0.093t}) = \ln(0.5)$

$\Rightarrow -0.093t \underbrace{\boxed{\ln e}}_{=1} = \ln 0.5 \Rightarrow t = \frac{\ln 0.5}{-0.093} \Rightarrow t =$ 7.45 years.

7. The half-life of a certain chemical in the human body for a health adult is approximately 5hr.

What is the exponential decay rate

k ⇒ Per hour

After 5 hours, $P(t) = \frac{1}{2}P_0 \Rightarrow \cancel{P_0} e^{-k(5)} = \frac{1}{2}\cancel{P_0}$

$\Rightarrow e^{-5k} = 0.5 \Rightarrow \ln e^{-5k} = \ln 0.5 \Rightarrow -5k = \ln 0.5$

$\Rightarrow k = \frac{\ln 0.5}{-5} \Rightarrow k \simeq 0.14$

How long will it take 93% of the chemical consumed to leave the body?

$\Rightarrow P(t) = P_0 e^{-0.14t}$

Chemical consumed $= \frac{93}{100} P_0 = 0.93 P_0$

$\Rightarrow$ left $= P_0 - 0.93P_0 = 0.07 P_0$

t for which $P(t) = 0.07 P_0$

$$\Rightarrow \cancel{P_0} e^{-0.14t} = 0.07 \cancel{P_0} \Rightarrow e^{-0.14t} = 0.07$$

$$\Rightarrow \ln e^{-0.14t} = \ln 0.07 \Rightarrow -0.14t = \ln 0.07 \Rightarrow t = \frac{\ln 0.07}{-0.14}$$

$$\Rightarrow t \cong 19 \text{ hrs.}$$

1. Researchers claim that a certain technology doubles in size every 6.9 years. What is the technology's exponential growth rate?

want to find k

$$P(t) = P_0 e^{kt} \quad \Rightarrow \quad P(6.9) = 2P_0$$

$$\Rightarrow \cancel{P_0} e^{k(6.9)} = 2\cancel{P_0}$$

$$\Rightarrow e^{6.9k} = 2 \quad \Rightarrow \quad \ln e^{6.9k} = \ln 2$$

$$\Rightarrow 6.9k = \ln 2 \quad \Rightarrow \quad k = \frac{\ln 2}{6.9} \quad \Rightarrow \quad k \cong 0.1$$

Quiz II

① Solve for x : $\log_x 100 = 2$

10 pts.

② If $\log x = 0.5$, $\log y = 2$, what is

10 pts

$\log \sqrt{x^2 y}$?