Math I 110 — Test #5 In-Class Review

1. Substitute the output of $f(x)$ into $g(x)$ and simplify. Given $f(x) = \sqrt{6x+5}$ and $g(x) = 3x - 4$, find $(g \circ f)(x)$.
2. Write $h(x)$ as $f(g(x))$. Find $f(x)$ and $g(x)$ such that $h(x) = (7x-2)^2$.
3. Determine whether $h(x) = 2x^3 - 5$ is one-to-one and find the inverse.
4. Determine whether $f(x) = 9x - 12$ is one-to-one and find the inverse.
5. Graph the function $y = 2^x - 4$.
6. Use $V(t) = 5200(0.83)^t$ to find the value after 10 years.
7. Simplify $\log_3\left(\frac{1}{27}\right)$.
8. Rewrite $\log_8(4) = x$ in exponential form.
9. Rewrite $6^2 = 36$ in logarithmic form.
10. Solve $\log_4\left(\frac{1}{16}\right) = x$.
11. Expand $\log(9y^7)$.
12. Expand $\log((2ab)^5)$.
13. Condense $\ln x - \ln 7 + 3\ln y$.
14. Expand $\ln\left(\frac{A^3B^6}{C^2}\right)$.
15. Evaluate $\log_7(15)$ and round to four decimals.
16. Given $\log(2) = 0.301$ and $\log(5) = 0.699$, find $\log_{25}(10)$.
17. Graph $f(x) = e^x$.
18. Solve $3^{x-2} = 27$.
19. Solve $4 + 6e^x = 52$.
20. The function $A(t) = 900e^{-0.183t}$. Find the time required for the substance to decay to half.

1. Substitute the output of $f(x)$ into $g(x)$ and simplify. Given $f(x) = \sqrt{6x+5}$ and $g(x) = 3x - 4$, find $(g \circ f)(x)$.

$$x \longrightarrow \boxed{f} \xrightarrow{\;y = f(x)\;} \boxed{g} \longrightarrow g(y) = g\circ f(x)$$

$$y = f(x) = \sqrt{6x+5}$$

$$g(y) = 3y - 4 = 3f(x) - 4$$

$$= 3\left(\sqrt{6x+5}\right) - 4$$

$$(g \circ f)(x) = 3\sqrt{6x+5} - 4$$

1a

$(f \circ g)(x)$

$$x \longrightarrow \boxed{g} \xrightarrow{\;y = g(x)\;} \boxed{f} \longrightarrow f(y) = (f\circ g)(x)$$

$$y = g(x) = 3x - 4$$

$$f(y) = \sqrt{6y+5}$$

$$= \sqrt{6(3x-4)+5}$$

$$(f\circ g)(x) = \sqrt{18x - 24 + 5} = \sqrt{18x - 19}$$

2. Write $h(x)$ as $f(g(x))$. Find $f(x)$ and $g(x)$ such that $h(x) = (7x-2)^2$.

$$x \longrightarrow \boxed{y = 7x-2} \xrightarrow{\;7x-2\;} \boxed{\text{Squares the input}} \xrightarrow{\;(7x-2)^2\;} = f(g(x))$$

$g(x)$ $\qquad$ $f(x)$

$$g(x) = 7x - 2 \quad \text{and} \quad f(x) = x^2$$

Alternatively

$f(x) = x, \quad g(x) = (7x-2)^2$

OR

$f(x) = (7x-2)^2, \quad g(x) = x$

3. Determine whether $h(x) = 2x^3 - 5$ is one-to-one and find the inverse.

Yes one-to-one. (Cubic function)

- $y = h(x)$
- Replace x and y
- Solve for y
- Replace y with $f^{-1}(x)$

Step 1 $y = 2x^3 - 5$

Step 2 $x = 2y^3 - 5 \Rightarrow 2y^3 - 5 = x$

Step 3 $\Rightarrow 2y^3 = x + 5 \Rightarrow y^3 = \frac{1}{2}(x+5)$

$\Rightarrow y = \sqrt[3]{\frac{1}{2}(x+5)}$

Step 4 $\Rightarrow f^{-1}(x) = \sqrt[3]{\frac{1}{2}(x+5)}$

4. Determine whether $f(x) = 9x - 12$ is one-to-one and find the inverse.

Yes, one-to-one. (Passes horizontal line test).

Step 1 $y = 9x - 12$

Step 2 $x = 9y - 12$

Step 3 $9y = x + 12 \Rightarrow y = \frac{1}{9}(x+12)$

Step 4 $f^{-1}(x) = \frac{1}{9}(x+12)$

5. Graph the function $y = 2^x - 4$.

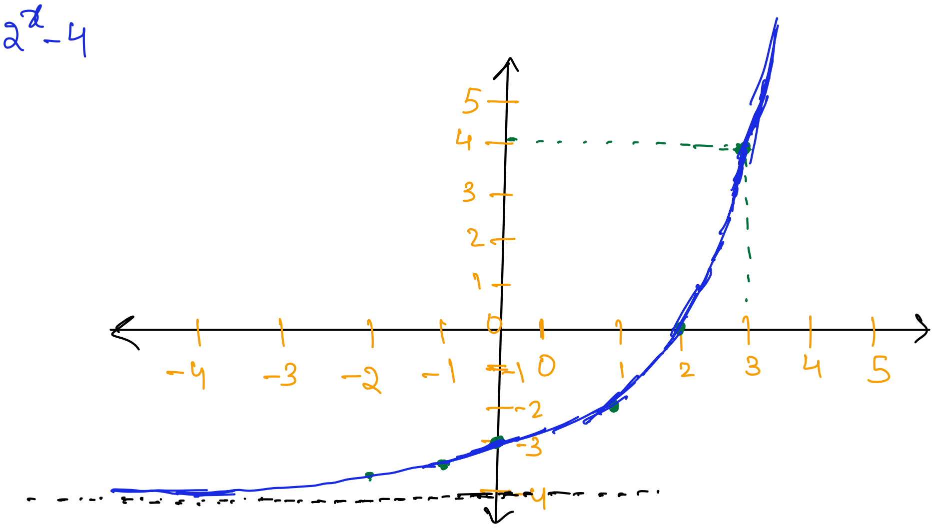
6. Use $V(t) = 5200(0.83)^t$ to find the value after 10 years.

7. Simplify $\log_3\left(\frac{1}{27}\right)$.

8. Rewrite $\log_8(4) = x$ in exponential form.

(5)

x	$y = 2^x - 4$
0	-3
1	-2
2	0
3	4
-1	-3.5
-2	-3.75



(6) $V(t) = 5200(0.83)^t$

<u>Value after 10 years.</u> $V(10) = 806.83$

(7) $\log_3\left(\frac{1}{27}\right) = x \Rightarrow 3^x = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$

$\Rightarrow x = -3$

(8) $\log_8 4 = x$. <u>Rewrite in exponential form.</u>

$\Rightarrow 8^x = 4$

9. Rewrite $6^2 = 36$ in logarithmic form. $\log_6 36 = 2$

10. Solve $\log_4\left(\frac{1}{16}\right) = x$. $\Rightarrow 4^x = \frac{1}{4^2} \Rightarrow 4^x = 4^{-2} \Rightarrow x = -2$

11. Expand $\log(9y^7)$. $= \log(9) + \log(y^7) = \log(9) + 7\log(y)$

12. Expand $\log((2ab)^5)$. $= 5\log(2ab) = 5(\log 2 + \log a + \log b)$
$= 5\log 2 + 5\log a + 5\log b$

13. Condense $\ln x - \ln 7 + 3\ln y$.

$\ln x - \ln 7 + \ln y^3 = \ln\frac{x}{7} + \ln y^3 = \ln\left(\frac{xy^3}{7}\right)$

14. Expand $\ln\left(\frac{A^3B^6}{C^2}\right)$.

$= \ln(A^3B^6) - \ln(C^2) = \ln(A^3) + \ln(B^6) - \ln(C^2)$
$= 3\ln A + 6\ln B - 2\ln C$

15. Evaluate $\log_7(15)$ and round to four decimals. $\log_7 15 = \frac{\log 15}{\log 7} = \frac{\ln 15}{\ln 7} = 1.3916$

16. Given $\log(2) = 0.301$ and $\log(5) = 0.699$, find $\log_{25}(10)$.

17. Graph $f(x) = e^x$. Done (Lecture 9.6)

18. Solve $3^{x-2} = 27$. $\Rightarrow 3^{x-2} = 3^3 \Rightarrow x - 2 = 3 \Rightarrow x = 5$

19. Solve $4 + 6e^x = 52$. $\Rightarrow 6e^x = 52 - 4 \Rightarrow 6e^x = 48 \Rightarrow e^x = 8 \Rightarrow \ln(e^x) = \ln(8)$

$x \ln e = \ln(8)$
$\Rightarrow x = \ln 8 = 2.0794$

20. The function $A(t) = 900e^{-0.183t}$. Find the time required for the substance to decay to half.

finding half-life

HW: See Lecture 9.7

16. Given $\log(2) = 0.301$ and $\log(5) = 0.699$, find $\log_{25}(10)$.

$$\log_{25} 10 = \frac{\log 10}{\log 25} = \frac{\log 2 + \log 5}{2\log 5} = \frac{0.301 + 0.699}{2(0.699)}$$

$$= 0.545$$

Answer Key (Click if interactive; otherwise scroll)

1. $3\sqrt{6x+5}-4$
2. $f(x)=x^2,\ g(x)=7x-2$
3. $h^{-1}(x)=\sqrt[3]{\dfrac{x+5}{2}}$
4. $f^{-1}(x)=\dfrac{x+12}{9}$
5. Key points: $(0,-3),(1,-2),(-1,-3.5)$; asymptote $y=-4$
6. $V(10)\approx 848.99$
7. -3
8. $8^x=4$
9. $\log_6(36)=2$
10. $x=-2$
11. $\log 9+7\log y$
12. $5\log 2+5\log a+5\log b$
13. $\ln\left(\dfrac{xy^3}{7}\right)$
14. $3\ln A+6\ln B-2\ln C$
15. ≈ 1.3010
16. ≈ 0.715
17. Key points: $(0,1),(1,e),(-1,1/e)$; asymptote $y=0$
18. $x=5$
19. $x\approx 2.0794$
20. $t\approx 3.79$ years