Problem 1: Reduce the following equations to one of the standard forms, classify the surface, and sketch it.

1. $4x^2 + y + 2z^2 = 0$

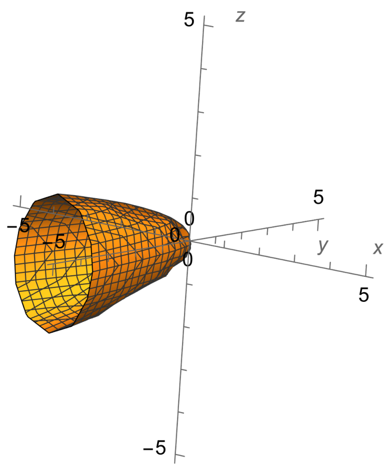
   Solution: $4x^2 + 2z^2 = -y$.

   The $x = k$ traces are: $4k^2 + 2z^2 = -y \Rightarrow 2z^2 = -(y + 4k^2)$ which are parabolas in the $x = k$ plane for any value of k. These parabolas have axis to be y-axis and they open towards the negative direction of y-axis.

   The $z = k$ traces are: $4x^2 + 2k^2 = -y \Rightarrow 4x^2 = -(y + 2k^2)$ which are parabolas in the $z = k$ plane for any value of k, having axis to be y-axis and opening towards the negative direction of y-axis.

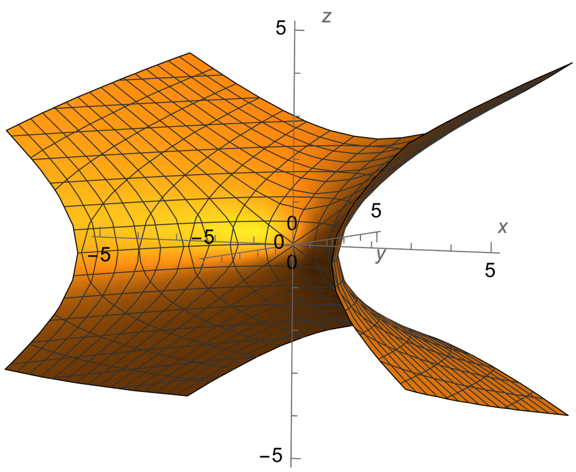
   The $y = k$ traces are: $4x^2 + 2z^2 = -k$ which are ellipses for $k < 0$.

   Thus the given equation represents an $\boxed{\text{elliptic paraboloid}}$ with axis being y-axis and opening towards the negative y-axis.



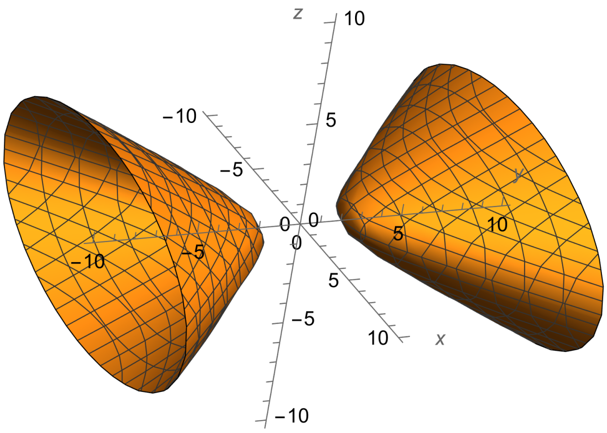
2. $x^2 + 2y - 2z^2 = 0$

   Solution: $x^2 + 2y - 2z^2 = 0 \Rightarrow 2y = 2z^2 - x^2 \Rightarrow y = z^2 - \dfrac{x^2}{2}$ which is the standard form of a $\boxed{\text{hyperbolic paraboloid}}$ with axis being the y-axis.



3. $y^2 = x^2 + 4z^2 + 4$

Solution: $y^2 = x^2 + 4z^2 + 4 \Rightarrow y^2 - x^2 - 4z^2 = 4 \Rightarrow -\dfrac{x^2}{4} + \dfrac{y^2}{4} - \dfrac{z^2}{1} = 1$ which is the standard form of a $\boxed{\text{hyperboloid with two sheets}}$ whose axis is the y-axis.



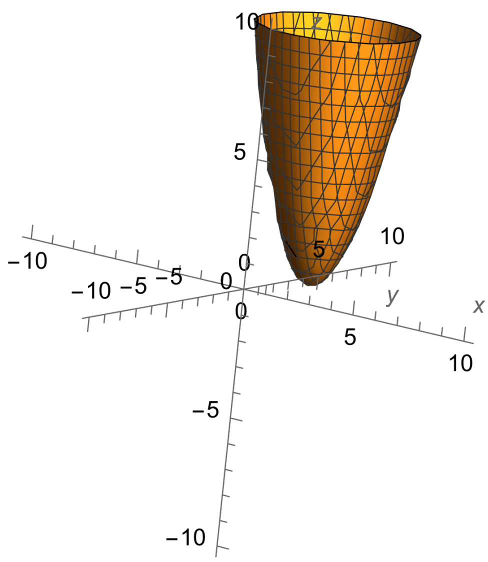
4. $x^2 + y^2 - 2x - 6y - z + 10 = 0$

Solution: $x^2 + y^2 - 2x - 6y - z + 10 = 0 \Rightarrow \underbrace{x^2 - 2x + 1}_{(x-1)^2} + \underbrace{y^2 - 6y + 9}_{(y-3)^2} - z = 0$

$\Rightarrow (x-1)^2 + (y-3)^2 - z = 0$

$$\Rightarrow z = (x-1)^2 + (y-3)^2$$

which is the standard form of an $\boxed{\text{elliptic paraboloid}}$ with vertex at $(1,3,0)$, axis being parallel to the z-axis and opening towards the +ve side of z-axis.



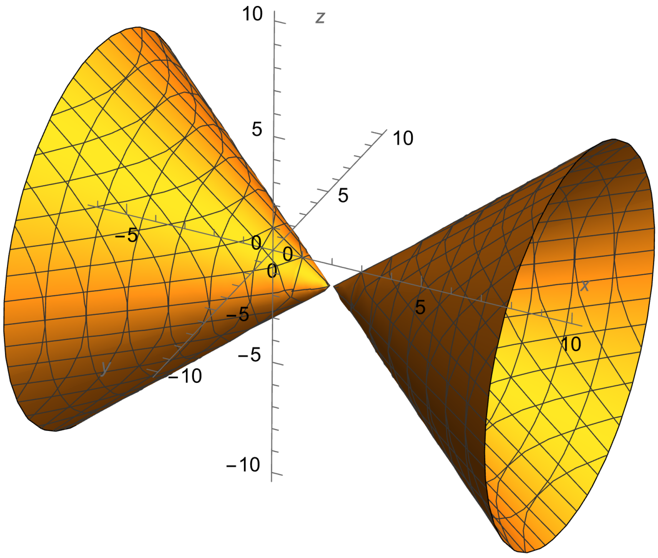
5. $x^2 - y^2 - z^2 - 4x - 2z + 3 = 0$

Solution: $x^2 - y^2 - z^2 - 4x - 2z + 3 = 0 \Rightarrow (\underbrace{x^2 - 4x + 4}_{(x-2)^2} - 4) - y^2 - (\underbrace{z^2 + 2z + 1}_{(z+1)^2} - 1) + 3 = 0$

$\Rightarrow (x-2)^2 - y^2 - (z+1)^2 = 0$

$$\Rightarrow (x-2)^2 = y^2 + (z+1)^2$$

which is standard form of a $\boxed{\text{cone}}$ centered at $(2, 0, -1)$ with axis parallel to the x-axis.



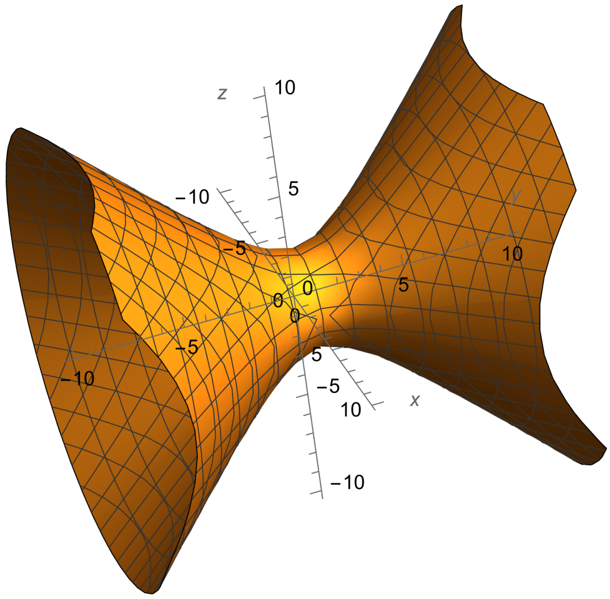
6. $x^2 - y^2 + z^2 - 4x - 2z = 0$

Solution: $x^2 - y^2 + z^2 - 4x - 2z = 0 \Rightarrow (\underbrace{x^2 - 4x + 4}_{(x-2)^2} - 4) - y^2 + (\underbrace{z^2 - 2z + 1}_{(z-1)^2} - 1) = 0$

$\Rightarrow (x-2)^2 - y^2 + (z-1)^2 = 5$

$$\Rightarrow \frac{(x-2)^2}{5} - \frac{y^2}{5} + \frac{(z-1)^2}{5} = 1$$

which is the standard form of a $\boxed{\text{hyperboloid of one sheet}}$ centered at $(2, 0, 1)$ whose axis is parallel to the y-axis.

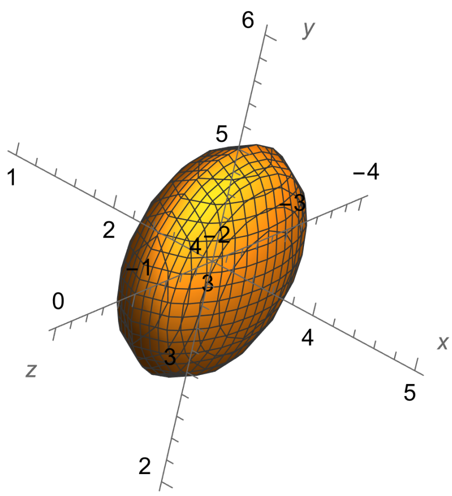


7. $4x^2 + y^2 + z^2 - 24x - 8y + 4z + 55 = 0$

Solution: $4x^2 + y^2 + z^2 - 24x - 8y + 4z + 55 = 0$
$\Rightarrow 4(\underbrace{x^2 - 6x + 9}_{(x-3)^2} - 9) + (\underbrace{y^2 - 8y + 16}_{(y-4)^2} - 16) + (\underbrace{z^2 + 4z + 4}_{(z+2)^2} - 4) + 55 = 0$

$$\Rightarrow 4(x-3)^2 + (y-4)^2 + (z+2)^2 = 1$$

which is the standard form of an $\boxed{\text{ellipsoid}}$ centered at $(3, 4, -2)$.

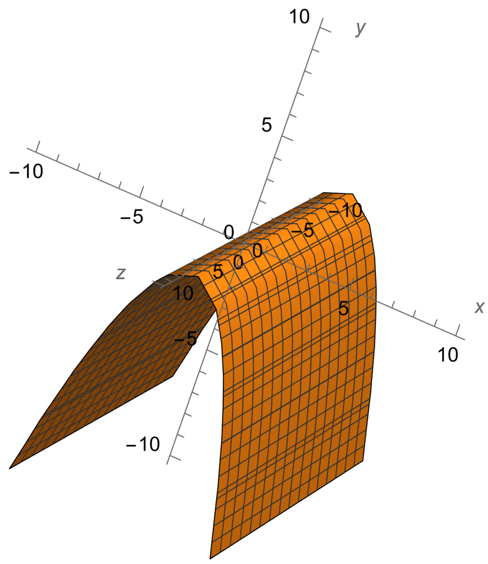


8. $x^2 - 2x + 2y - 1 = 0$

Solution: $x^2 - 2x + 2y - 1 = 0 \Rightarrow (\underbrace{x^2 - 2x + 1}_{(x-1)^2} - 1) + 2y - 1 = 0$

$$\Rightarrow (x-1)^2 = -2(y-1)$$

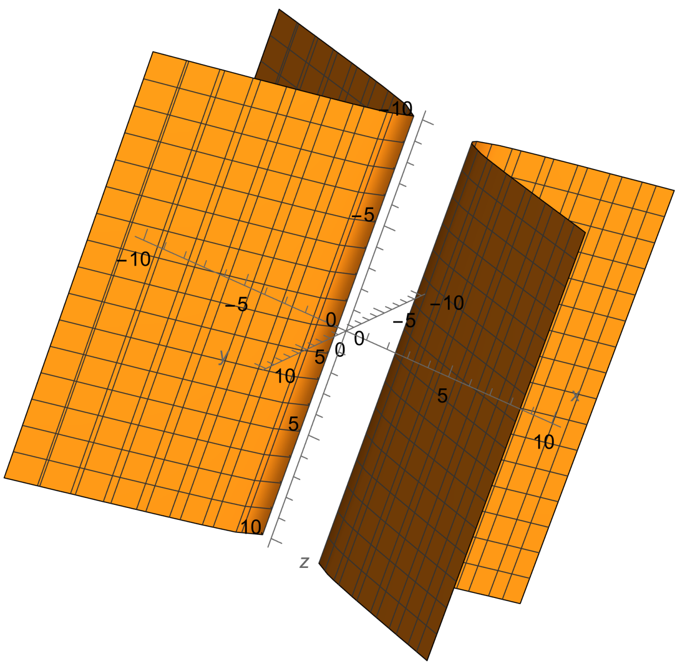
which represents a $\boxed{\text{parabolic cylinder}}$ which open toward the negative y-axis and whose axis is the line parallel to z-axis passing through $(1, 1, 0)$.



Note that if the equation was $x^2 - 2x - 2y^2 - 1 = 0$, then after completion of squares we get $(x-1)^2 - 2y^2 = 2$ or

$$\frac{(x-1)^2}{2} - \frac{y^2}{1} = 1$$

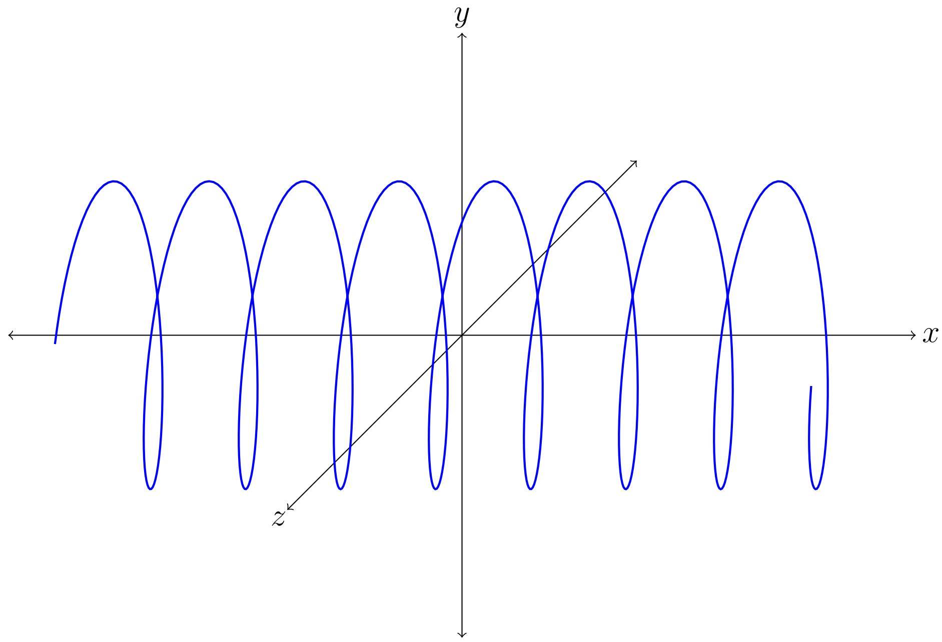
which represents a $\boxed{\text{hyperbolic cylinder}}$ whose axis is the line parallel to z-axis and passing through the point $(1, 0, 0)$.



Problem 2: Sketch the following curves.

1. $\vec{r}\,(t) = t\,\hat{i} + 2\sin t\,\hat{j} + \cos t\,\hat{k}$

   Solution: The equation represents an elliptical helix whose axis is the x-axis



2. $\vec{r}\,(t) = 2\cos t\,\hat{i} + t\,\hat{j} + \sin t\,\hat{k}$

   Solution: The equation represents an elliptical helix whose axis is the y-axis.