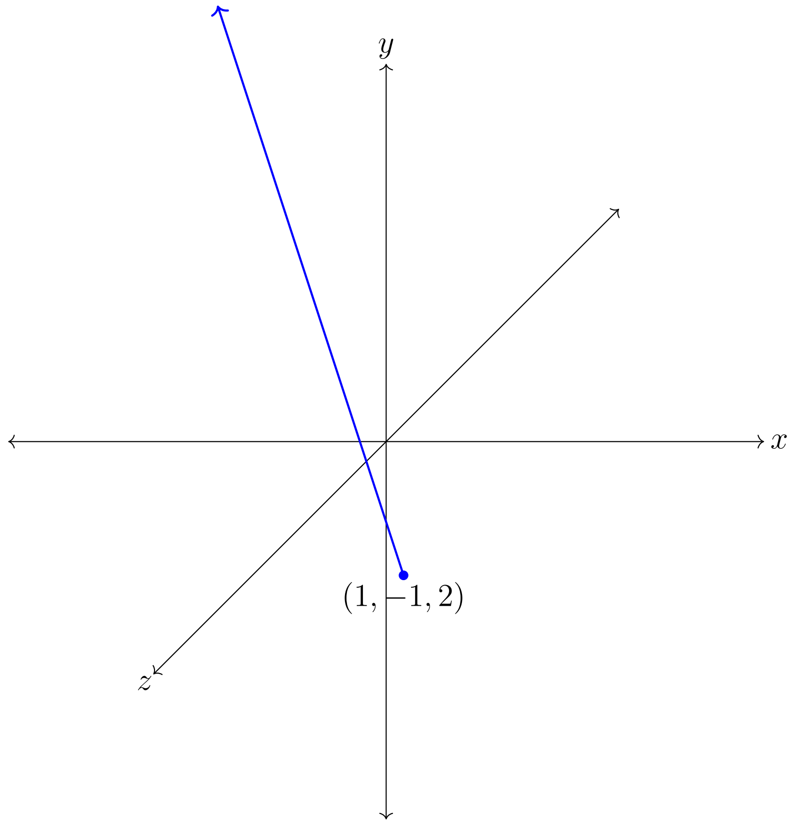
Problem 1: Sketch the following curves.

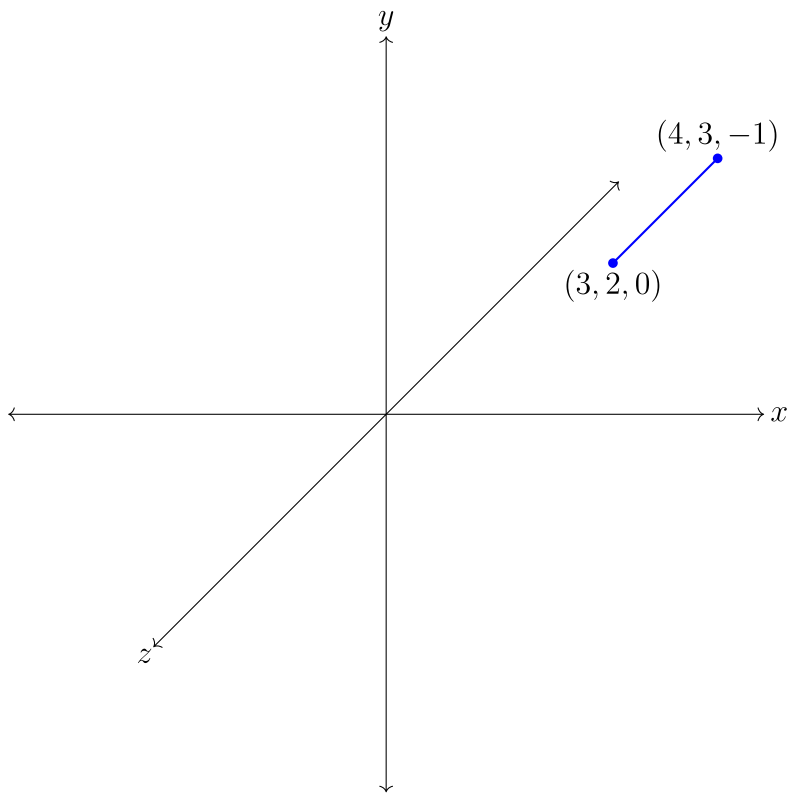
1. $\vec{r}\,(t) = t\,\hat{i} + (2-t)\,\hat{j} + (1+t)\,\hat{k}$, $t \le 1$

   Solution: The given curve is a ray starting from $(1,-1,2)$ in the direction $-\hat{i}+\hat{j}-\hat{k}$.



2. $\vec{r}\,(t) = (2+t)\,\hat{i} + (1+t)\,\hat{j} + (1-t)\,\hat{k}$, $1 \le t \le 2$.

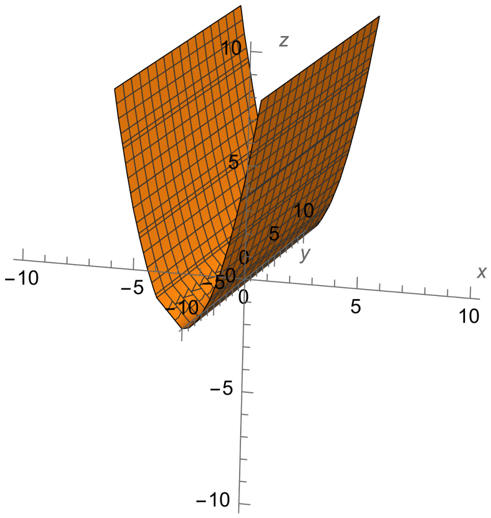
   Solution: The given curve is a line segment joining the points $(3,2,0)$ and $(4,3,-1)$



Problem 2: Sketch the graphs of the following functions of two variables. Use the knowledge quadric surfaces if needed.

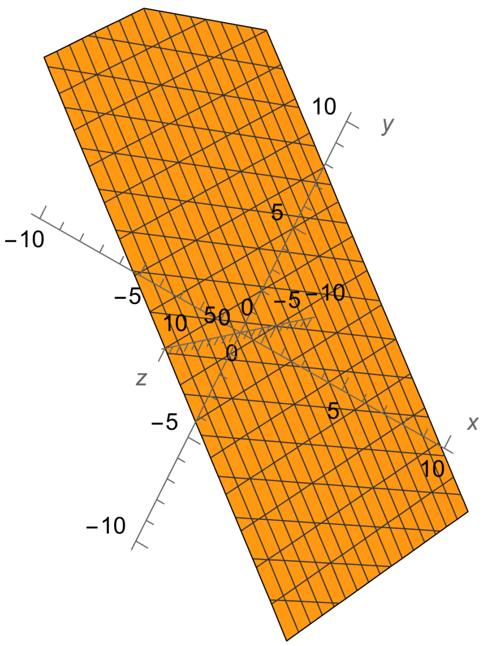
1. $f(x, y) = x^2$

   Solution: The graph is given by $z = x^2$ which is a parabolic cylinder whose axis is y-axis.



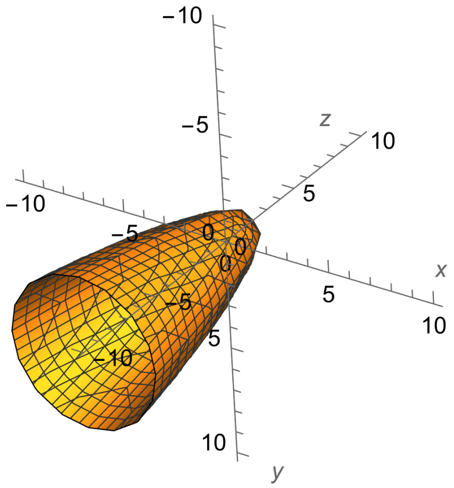
2. $f(x, y) = 10 - 4x - 5y$

   Solution: The graph is given by $z = 10 - 4x - 5y$ which is a plane.



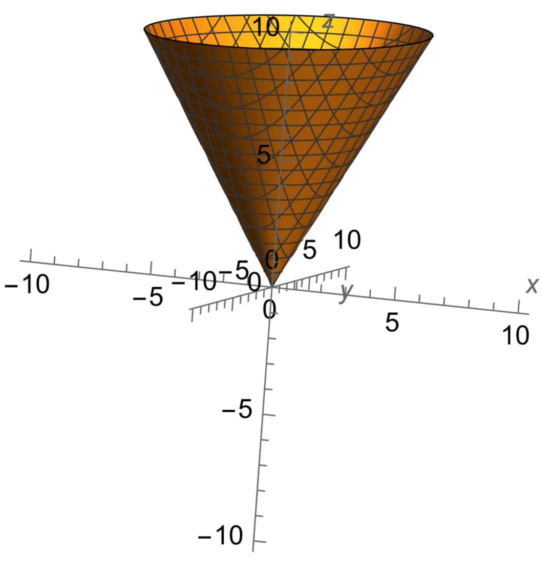
3. $f(x, y) = 2 - x^2 - y^2$

   Solution: The graph is given by $z = 2 - x^2 - y^2$ or $-(z-2) = x^2 + y^2$ which is an elliptic paraboloid with vertex at $(0, 0, 2)$ and axis being -ve z-axis.



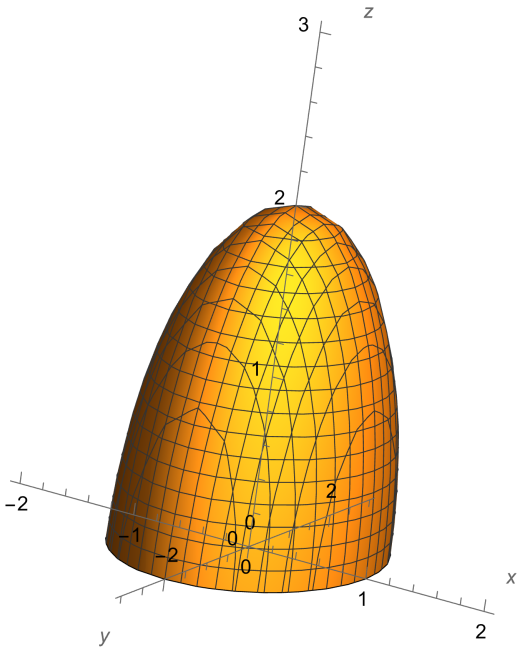
4. $f(x, y) = \sqrt{4x^2 + y^2}$

   Solution: The graph is given by $z^2 = 4x^2 + y^2$ and $z > 0$ which is the upper part of a cone with vertex at $(0, 0, 0)$ and axis being the z-axis.



5. $f(x,y) = \sqrt{4 - 4x^2 - y^2}$

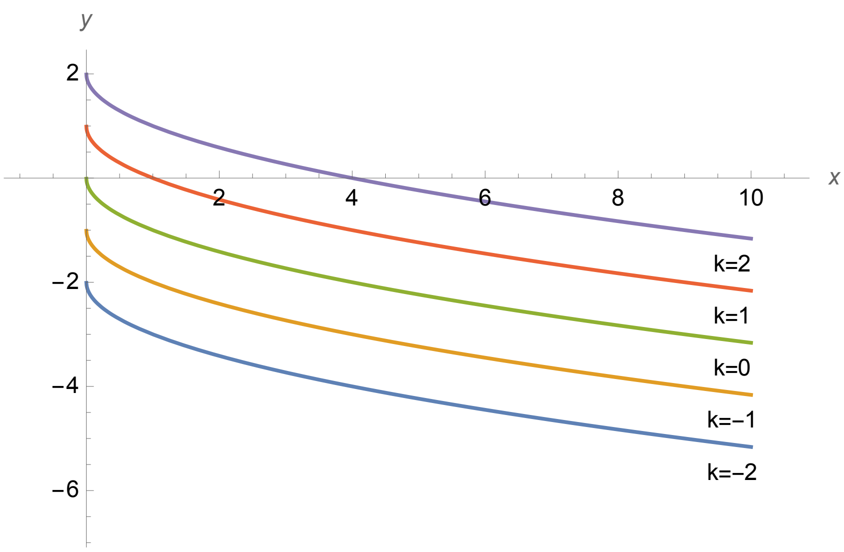
   Solution: The graph is given by $4x^2 + y^2 + z^2 = 4$ and $z > 0$ which is the upper part of an ellipsoid centered at $(0,0,0)$.



Problem 3: Sketch the level curves (also called contour curves) of the following functions of two variables for the level values $k = -2, -1, 0, 1, 2$.

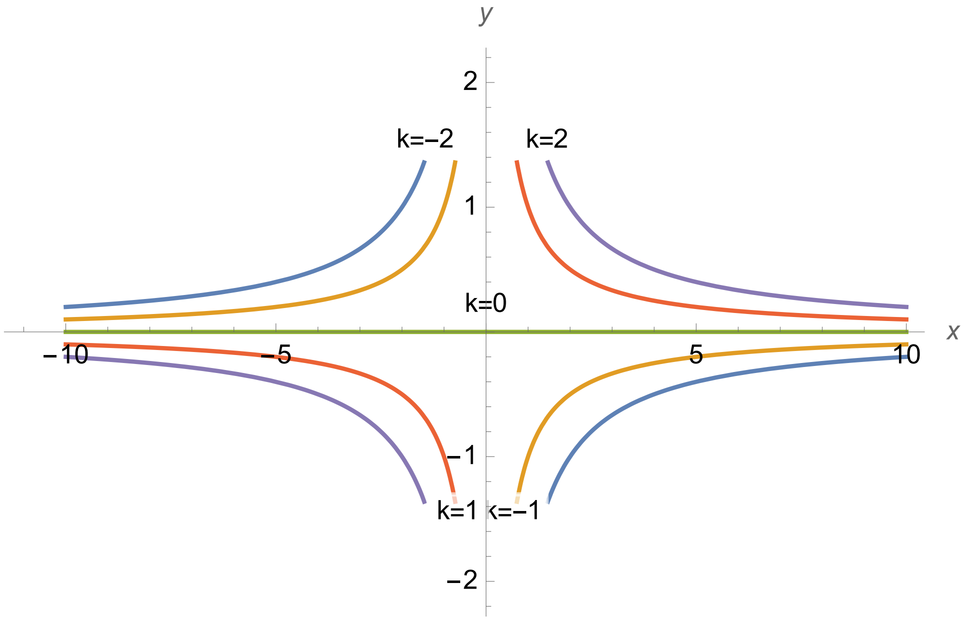
1. $f(x,y) = \sqrt{x} + y$

   Solution: The level curves are $y = k - \sqrt{x}$ for various values of k.



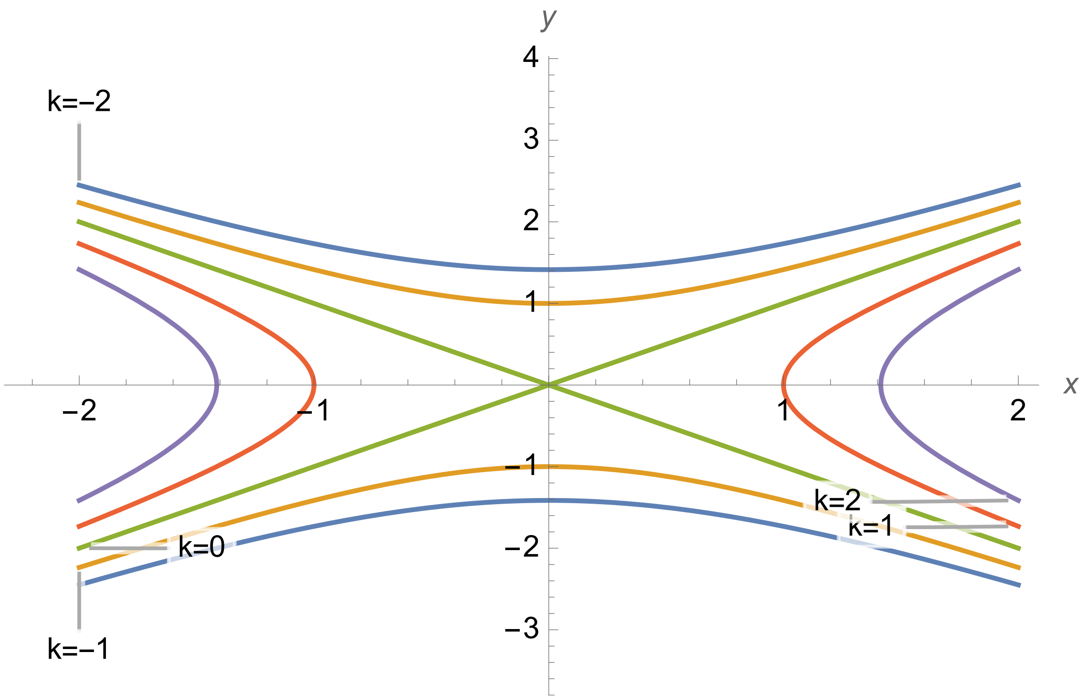
2. $f(x, y) = xy$

   Solution: The level curves are $y = \dfrac{k}{x}$ for various values of k.



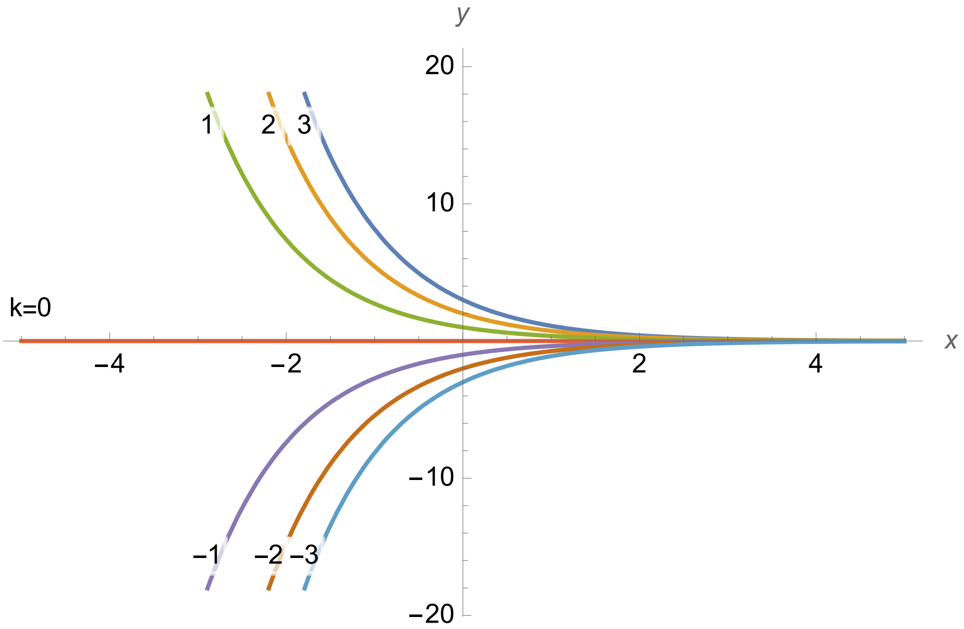
3. $f(x, y) = x^2 - y^2$

   Solution: The level curves are $x^2 - y^2 = k$ for various values of k.



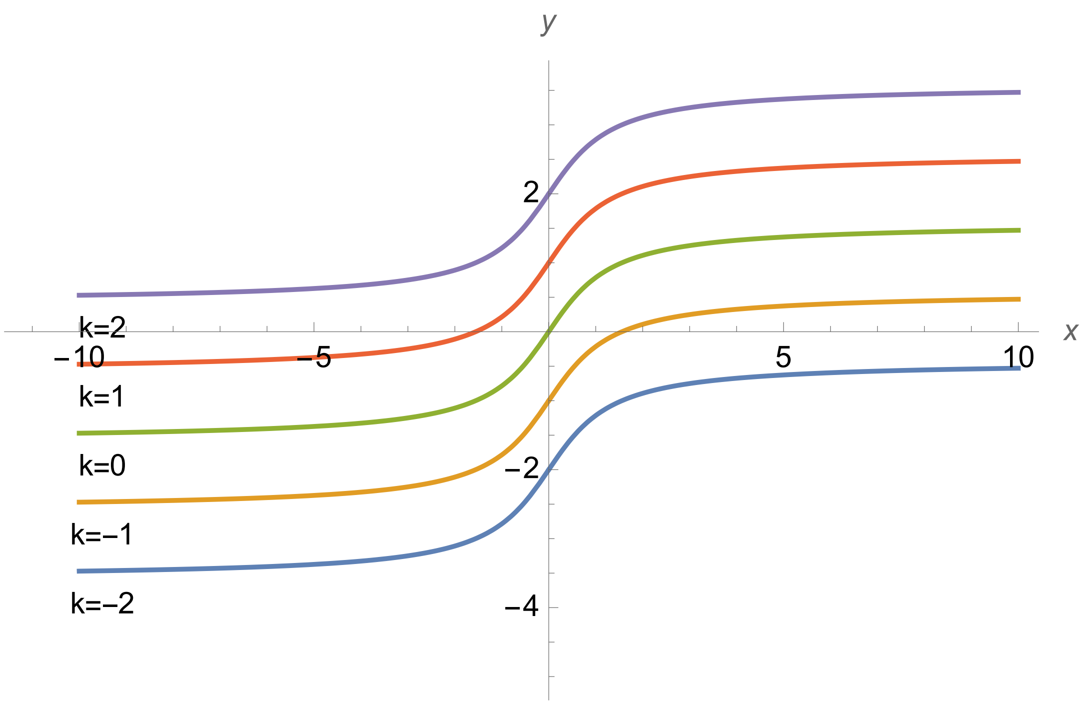
4. $f(x, y) = ye^x$

Solution: The level curves are $k = ye^x$ or $y = ke^{-x}$.



5. $f(x, y) = y - \tan^{-1}(x)$

Solution: The level curves are $y = k + \tan^{-1}(x)$ for various values of k.



Problem 4: Convert the following Cartesian coordinates into cylindrical and spherical coordinates.

$$(-1,1,1) \quad (-\sqrt{2},\sqrt{2},1) \quad (1,0,\sqrt{3}) \quad (\sqrt{3},-1,2\sqrt{3})$$

Solution: For $(-1,1,1)$:-

$r = \sqrt{x^2+y^2} = \sqrt{(-1)^2+(1)^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{-1}\right) = \frac{3\pi}{4}$

$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{(-1)^2+(1)^2+(1)^2} = \sqrt{3}$

$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \tan^{-1}\left(\frac{\sqrt{2}}{1}\right) = \tan^{-1}(\sqrt{2})$

Thus, the cylinderical coordinates are $(\sqrt{2}, 3\pi/4, 1)$ and spherical coordinates are $\big(\sqrt{3}, 3\pi/4, \tan^{-1}(\sqrt{2})\big)$.

For $(-\sqrt{2},\sqrt{2},1)$:-

$r = \sqrt{x^2+y^2} = \sqrt{(-\sqrt{2})^2+(\sqrt{2})^2} = 2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right) = \frac{3\pi}{4}$

$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{(-\sqrt{2})^2+(\sqrt{2})^2+(1)^2} = \sqrt{5}$

$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \tan^{-1}\left(\frac{2}{1}\right) = \tan^{-1}(2)$

Thus, the cylinderical coordinates are $(2, 3\pi/4, 1)$ and spherical coordinates are $\big(\sqrt{5}, 3\pi/4, \tan^{-1}(2)\big)$.

For $(1,0,\sqrt{3})$:-

$r = \sqrt{x^2+y^2} = \sqrt{(1)^2+(0)^2} = 1$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{(1)^2+(0)^2+(\sqrt{3})^2} = 2$

$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

Thus, the cylinderical coordinates are $(1, 0, \sqrt{3})$ and spherical coordinates are $\big(2, 0, \pi/6\big)$.

For $(\sqrt{3},-1,2\sqrt{3})$:-

$r = \sqrt{x^2+y^2} = \sqrt{(\sqrt{3})^2+(-1)^2} = 2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{11\pi}{6}$

$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{(\sqrt{3})^2+(-1)^2+(2\sqrt{3})^2} = 4$

$\phi = \tan^{-1}\left(\dfrac{\sqrt{x^2+y^2}}{z}\right) = \tan^{-1}\left(\dfrac{2}{2\sqrt{3}}\right) = \pi/6$

Thus, the cylinderical coordinates are $(2, 11\pi/6, 2\sqrt{3})$ and spherical coordinates are $\big(4, 11\pi/6, \pi/6\big)$.

Problem 5: Convert the following cylinderical coordinates into Cartesian and Spherical coordinates.

$$(4, \pi/3, -2) \quad (2, -\pi/2, 1) \quad (\sqrt{2}, 3\pi/4, 2)$$

Solution: For $(4, \pi/3, -2)$:-

$x = r\cos\theta = 4\cos(\pi/3) = 2$

$y = r\sin\theta = 4\sin(\pi/3) = 2\sqrt{3}$

$\rho = \sqrt{r^2+z^2} = \sqrt{(4)^2+(-2)^2} = 2\sqrt{5}$

$\phi = \tan^{-1}\left(\dfrac{r}{z}\right) = \tan^{-1}\left(\dfrac{4}{-2}\right) = \pi - \tan^{-1}(2)$

So, Cartesian coordinates are $(2, 2\sqrt{3}, -2)$ and spherical coordinates are $\big(2\sqrt{5}, \pi/3, \pi - \tan^{-1}(2)\big)$.

For $(2, -\pi/2, 1)$:-

$x = r\cos\theta = 2\cos(-\pi/2) = 0$

$y = r\sin\theta = 2\sin(-\pi/2) = -2$

$\rho = \sqrt{r^2+z^2} = \sqrt{(2)^2+(1)^2} = \sqrt{5}$

$\phi = \tan^{-1}\left(\dfrac{r}{z}\right) = \tan^{-1}\left(\dfrac{2}{1}\right) = \tan^{-1}(2)$

So, Cartesian coordinates are $(0, -2, 1)$ and spherical coordinates are $\big(\sqrt{5}, -\pi/2, \tan^{-1}(2)\big)$.

For $(\sqrt{2}, 3\pi/4, 2)$:-

$x = r\cos\theta = \sqrt{2}\cos(3\pi/4) = -1$

$y = r\sin\theta = \sqrt{2}\sin(3\pi/4) = 1$

$\rho = \sqrt{r^2+z^2} = \sqrt{(\sqrt{2})^2+(2)^2} = \sqrt{6}$

$\phi = \tan^{-1}\left(\dfrac{r}{z}\right) = \tan^{-1}\left(\dfrac{\sqrt{2}}{2}\right) = \tan^{-1}(1/\sqrt{2})$

So, Cartesian coordinates are $(-1, 1, 2)$ and spherical coordinates are $\big(\sqrt{6}, 3\pi/4, \tan^{-1}(1/\sqrt{2})\big)$.

Problem 6: Convert the following spherical coordinates into Cartesian and cylinderical coordinates.

$$(6, \pi/3, \pi/6) \quad (3, \pi/2, 3\pi/4) \quad (2, \pi/2, \pi/2)$$

Solution: For $(6, \pi/3, \pi/6)$:-

$x = \rho \cos\theta \sin\phi = 6\cos(\pi/3)\sin(\pi/6) = 3/2$

$y = \rho \sin\theta \sin\phi = 6\sin(\pi/3)\sin(\pi/6) = 3\sqrt{3}/2$

$z = \rho \cos\phi = 6\cos(\pi/6) = 3\sqrt{3}$

$r = \rho \sin\phi = 6\sin(\pi/6) = 3$

So, cylindrical coordinates are $(3, \pi/3, 3\sqrt{3})$ and Cartesian coordinates are $\left(3/2, 3\sqrt{3}/2, 3\sqrt{3}\right)$.

For $(3, \pi/2, 3\pi/4)$:-

$x = \rho \cos\theta \sin\phi = 3\cos(\pi/2)\sin(3\pi/4) = 0$

$y = \rho \sin\theta \sin\phi = 3\sin(\pi/2)\sin(3\pi/4) = 3/\sqrt{2}$

$z = \rho \cos\phi = 3\cos(3\pi/4) = -3/\sqrt{2}$

$r = \rho \sin\phi = 3\sin(3\pi/4) = 3/\sqrt{2}$

So, cylindrical coordinates are $(3/\sqrt{2}, \pi/2, -3/\sqrt{2})$ and Cartesian coordinates are $\left(0, 3/\sqrt{2}, -3/\sqrt{2}\right)$.

For $(2, \pi/2, \pi/2)$:-

$x = \rho \cos\theta \sin\phi = 2\cos(\pi/2)\sin(\pi/2) = 0$

$y = \rho \sin\theta \sin\phi = 2\sin(\pi/2)\sin(\pi/2) = 2$

$z = \rho \cos\phi = 2\cos(\pi/2) = 0$

$r = \rho \sin\phi = 2\sin(\pi/2) = 2$

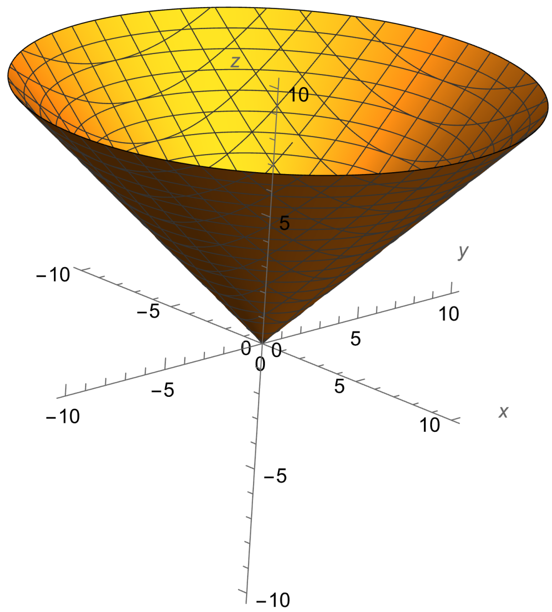
So, cylindrical coordinates are $(2, \pi/2, 0)$ and Cartesian coordinates are $(0, 2, 0)$.

Problem 7: Describe and sketch the surface whose equation in cylinderical coordinates is the following

1. $r = z$

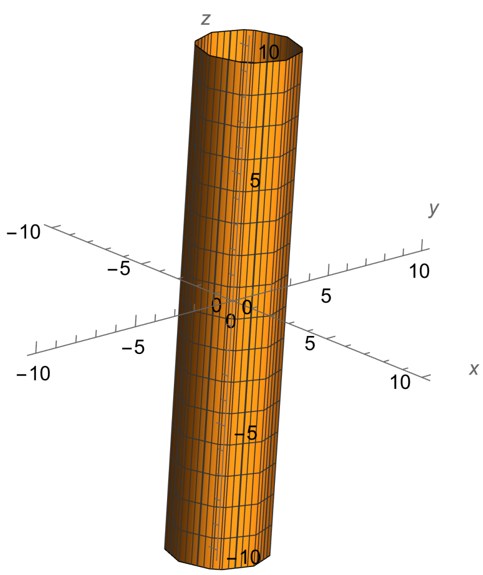
   Solution: Equation in Cartesian coordinates is: $\sqrt{x^2 + y^2} = z \Rightarrow x^2 + y^2 - z^2$ and $z > 0$.

   This represents the part lying above xy-plane, of a cone with vertex $(0, 0, 0)$ and axis being z-axis.



2. $r = 2$

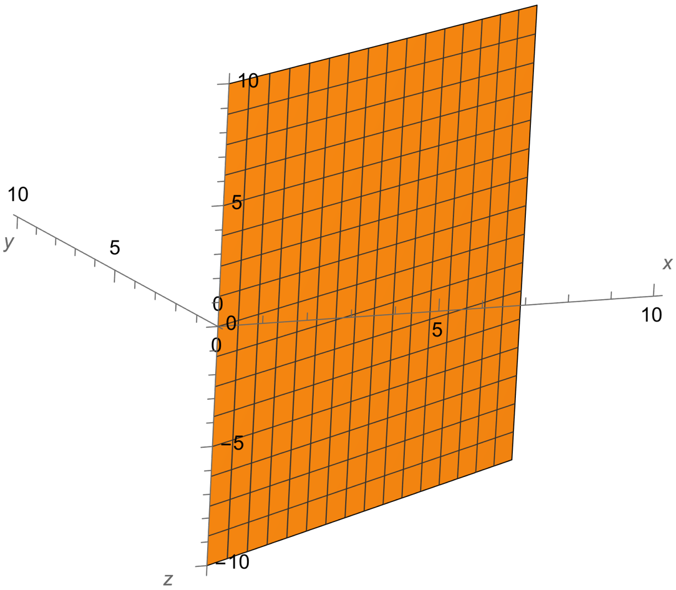
Solution: Equation in Cartesian coordinates is: $\sqrt{x^2 + y^2} = 2 \Rightarrow x^2 + y^2 = 4$ which is circular cylinder whose axis is the z-axis.



3. $\theta = \pi/6$

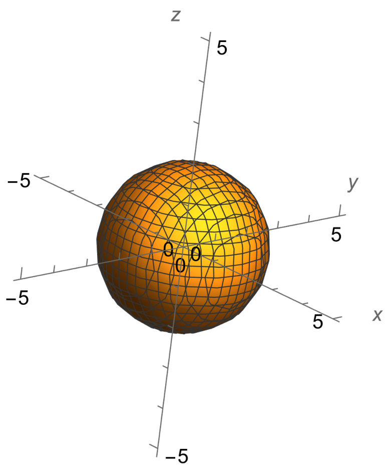
Solution: Equation in Cartesian coordinates is: $\tan^{-1}\left(\dfrac{y}{x}\right) = \dfrac{\pi}{6}$

$\Rightarrow \dfrac{y}{x} = \tan(\pi/6)$ and $x > 0$, $y > 0 \Rightarrow x - \sqrt{3}y = 0$, $x > 0$, $y > 0$ which is a half-plane.



4. $r^2 + z^2 = 4$

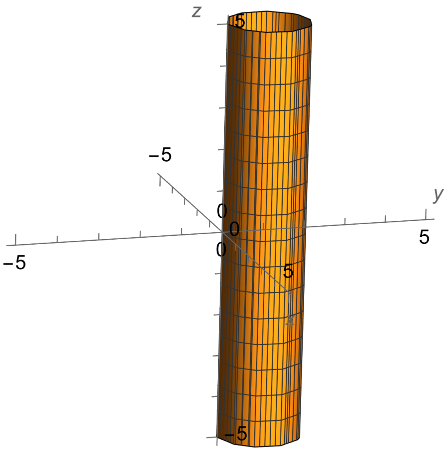
Solution: Equation in Cartesian coordinates is: $x^2 + y^2 + z^2 = 4$ which is sphere of radius 2 centered at $(0, 0, 0)$.



5. $r = 2\sin\theta$

Solution: Equation in Cartesian coordinates is: $\sqrt{x^2+y^2} = 2\dfrac{y}{\sqrt{x^2+y^2}}$

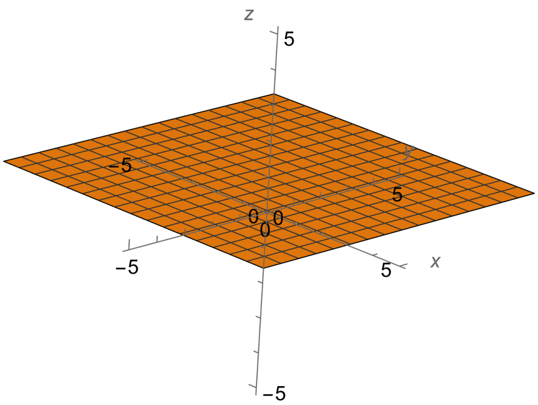
$\Rightarrow x^2 + y^2 - 2y = 0 \Rightarrow x^2 + (y-1)^2 = 1$ which is a circular cylinder with axis passing through $(0, 1, 0)$ and parallel to z-axis.



Problem 8: Describe and sketch the surface whose equation in spherical coordinates is the following

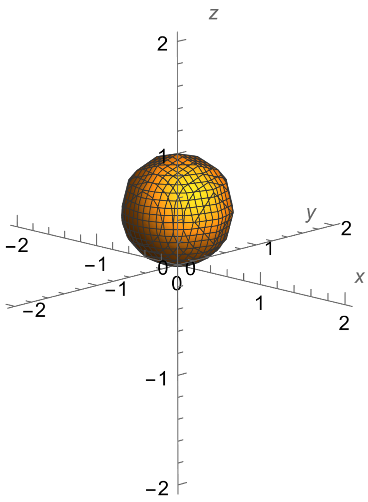
1. $\rho \cos \phi = 1$

   Solution: Equation in Cartesian coordinates is: $z = 1$ which is a plane whose normal vector is $\hat{k}$.



2. $\rho = \cos \phi$

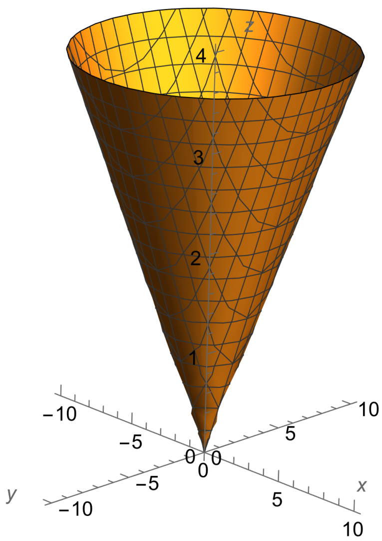
   Solution: Equation in Cartesian coordinates is: $\rho = \dfrac{z}{\rho} \Rightarrow \rho^2 = z \Rightarrow x^2 + y^2 + z^2 = z \Rightarrow$ $x^2 + y^2 + \left(z - \dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ which is a sphere of radius $1/2$ centered at $(0, 0, 1/2)$.



3. $\phi = \pi/3$

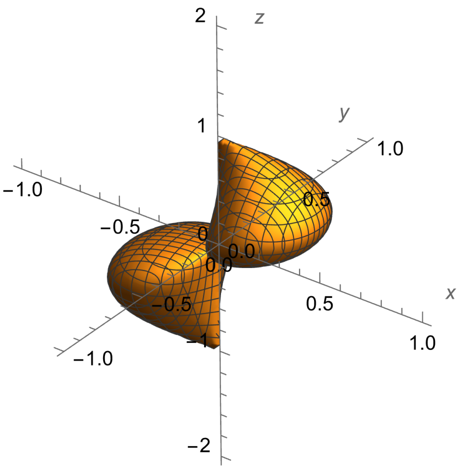
Solution: $\dfrac{z}{\sqrt{x^2+y^2+z^2}} = \cos(\pi/3) = \dfrac{1}{2} \Rightarrow x^2+y^2+z^2 = 4z^2$ and $z > 0$

$\Rightarrow x^2+y^2 = 3z^2$ and $z > 0$ which is the upper half (part above the xy-plane) of a cone with vertex at $(0,0,0)$ and axis being the z-axis.



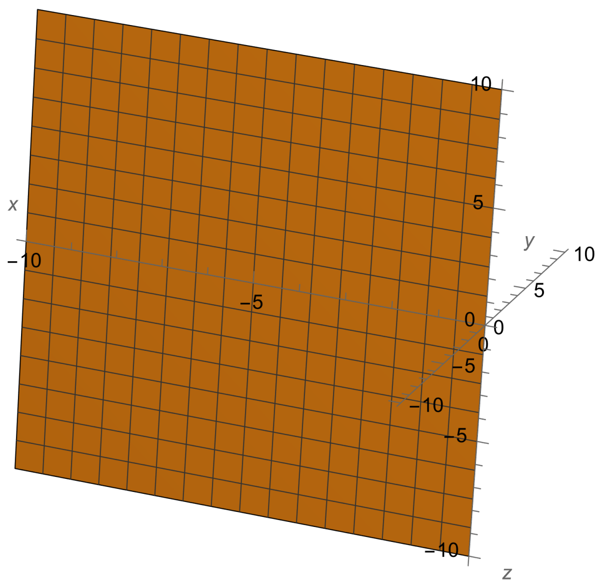
4. $\rho = \cos\theta\cos\phi$

Solution: $\sqrt{x^2+y^2+z^2} = \dfrac{x}{\sqrt{x^2+y^2}}\dfrac{z}{\sqrt{x^2+y^2+z^2}} \Rightarrow (x^2+y^2+z^2)\left(\sqrt{x^2+y^2}\right) = xz$ which is not a quadric surface. The surface is shown in the figure below:-



5. $\theta = \pi$

Solution: In Cartesian coordinates we have $\tan^{-1}\left(\dfrac{y}{x}\right) = \pi \Rightarrow y = 0$ and $x < 0$ which is a half-plane.



Problem 9: Write following Cartesian equations in cylinderical and spherical coordinates.

1. $x^2 - x + y^2 + z^2 = 1$

   Solution: In cylindrical coordinates we have:-

   $(r\cos\theta)^2 - (r\cos\theta) + (r\sin\theta)^2 + z^2 = 1 \Rightarrow r^2 - r\cos\theta + z^2 = 1$

   In spherical coordinates we have:-

   $(x^2 + y^2 + z^2) - x = 1 \Rightarrow \rho^2 - \rho\cos\theta\sin\phi = 1$

2. $z = x^2 - y^2$

   Solution: In cylindrical coordinates we have:-

   $z = (r\cos\theta)^2 - (r\sin\theta)^2 \Rightarrow z = r^2\cos 2\theta.$

   In spherical coordinates we have:-

   $\rho\cos\phi = (\rho\cos\theta\sin\phi)^2 - (\rho\sin\theta\sin\phi)^2 \Rightarrow \cos\phi = \rho\sin^2\phi\cos 2\theta.$

3. $z = x^2 + y^2$

   Solution: In cylindrical coordinates we have:-

   $z = (r\cos\theta)^2 + (r\sin\theta)^2 \Rightarrow z = r^2.$

   In spherical coordinates we have:-

   $\rho\cos\phi = (\rho\cos\theta\sin\phi)^2 + (\rho\sin\theta\sin\phi)^2 \Rightarrow \cos\phi = \rho\sin^2\phi.$

4. $x^2 - y^2 - z^2 = 1$

   Solution: In cylindrical coordinates we have:-

   $(r\cos\theta)^2 - (r\sin\theta)^2 - z^2 = 1 \Rightarrow r^2\cos 2\theta - z^2 = 1.$

   In spherical coordinates we have:-

   $(\rho\cos\theta\sin\phi)^2 - (\rho\sin\theta\sin\phi)^2 - (\rho\cos\phi)^2 = 1 \Rightarrow \rho^2\sin^2\phi\cos 2\theta - \rho^2\cos^2\phi = 1.$

Problem 10: Identify and Sketch the surfaces with the following parametric/vector equations.

1. $\vec{r}(u,v) = (u+v)\,\hat{i} + (3-v)\,\hat{j} + (1+4u+5v)\,\hat{k}$

   Solution: In parametric form we have:-

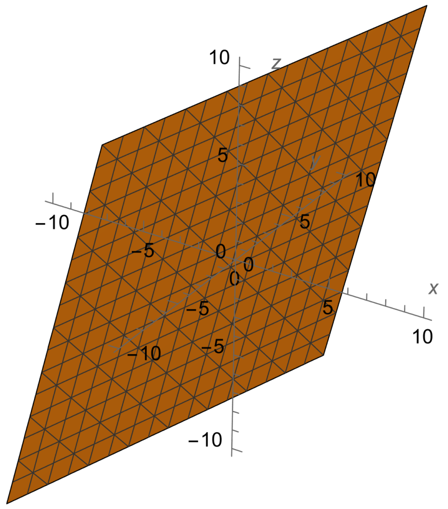
   $x(u,v) = u+v$, $y(u,v) = 3-v$, $z(u,v) = 1+4u+5v$

   Eliminate the parameters u and v to get the Cartesian equation:-

   $y = 3 - v \Rightarrow v = 3 - y$. Since $x = u + v$, we have $u = x - v = x - (3-y) = x + y - 3$.

   $z = 1 + 4u + 5v \Rightarrow z = 1 + 4(x+y-3) + 5(3-y) \Rightarrow z = 4x - y + 4$

   $\Rightarrow 4x - y - z + 4 = 0$ which is the equation of a plane as shown in the figure below:-

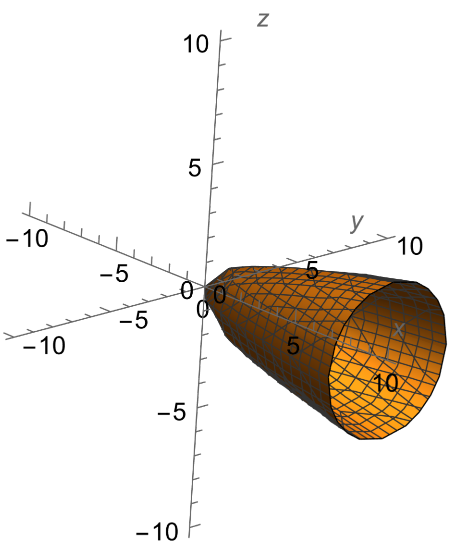


2. $x = u^2$, $y = u\cos v$, $z = u\sin v$

Solution: Eliminate the parameters u and v to get the Cartesian equation:-

$y^2 + z^2 = (u\cos v)^2 + (u\sin v)^2 = u^2(\cos^2 v + \sin^2 v) = u^2 = x$

Thus, the Cartesian equation of the given surface is $x = y^2 + z^2$ which an elliptic paraboloid with axis being the x-axis and vertex at $(0, 0, 0)$.

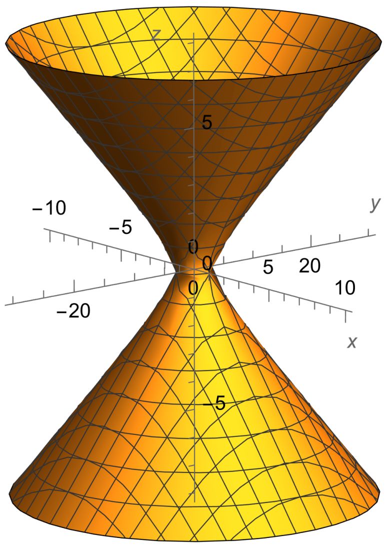


3. $x = (\cos t)(\sec s)\,,\ y = 3\,(\sin t)(\sec s)\,,\ z = \tan s$

Solution: Eliminate the parameters s and t to get the Cartesian equation:-

$x^2 + (y/3)^2 = \sec^2 s\,(\cos^2 t + \sin^2 t) = \sec^2 s \Rightarrow x^2 + (y/3)^2 - z^2 = \sec^2 s - \tan^2 s = 1$

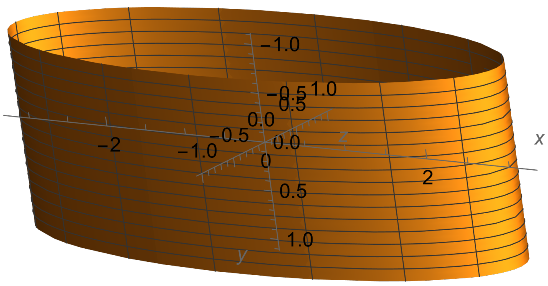
Thus, the Cartesian equation of the given surface is $x^2 + \dfrac{y^2}{9} - z^2 = 1$ which is a hyperboloid of one sheet whose axis is the z-axis.



4. $x = 3\,\cos t\,,\ y = s\,,\ z = \sin t\,,\ -1 \leq s \leq 1.$

Solution: Eliminate the parameters s and t to get the Cartesian equation:-

$(x/3)^2 + y^2 = \cos^2 t + \sin^2 t = 1 \Rightarrow \dfrac{x^2}{9} + y^2 = 1$ which is the equation of an elliptic cylinder whose axis is the z-axis. Since the parameter s has range limited to $-1 \leq s \leq 1$ and $z = s$, we have $-1 \leq z \leq 1$. Thus, the cylinder has finite height.



Problem 11: Find the parametric equation for the following surfaces

1. The plane through the origin that contains the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$.

   Solution: Vector equation of a plane passing through a point (x_0, y_0, z_0) and containing two non-parallel vectors $\vec{a}$ and $\vec{b}$ is given by:-

$$\vec{r}(u,v) = x_0\,\hat{\imath} + y_0\,\hat{\jmath} + z_0\,\hat{k} + u\,\vec{a} + v\,\vec{b}$$

   where u, v range over all real numbers.

   For this case the point is $(0,0,0)$, $\vec{a} = \hat{\imath} - \hat{\jmath}$ and $\vec{b} = \hat{\jmath} - \hat{k}$. Thus, we have

$$\vec{r}(u,v) = u\,(\hat{\imath} - \hat{\jmath}) + v\,(\hat{\jmath} - \hat{k}) = u\,\hat{\imath} + (v-u)\,\hat{\jmath} - v\,\hat{k}$$

   Therefore, the parametric equation of the given plane is given by:-

$$\boxed{x = u\,, \quad y = v - u\,, \quad z = -v}$$

2. The part of the hyperboloid $4x^2 - 4y^2 - z^2 = 4$ that lies in front of the yz-plane.

   Solution: The given equation is $\dfrac{x^2}{1} - \dfrac{y^2}{1} - \dfrac{z^2}{4} = 1$ or $\dfrac{x^2}{1} - \left(\dfrac{y^2}{1} + \dfrac{z^2}{4}\right) = 1$

   Using $\sec^2\phi - \tan^2\phi = 1$ we let $x = \sec\phi$ and $\dfrac{y^2}{1} + \dfrac{z^2}{4} = \tan^2\phi$.

   Using $\sin^2\theta + \cos^2\theta = 1$ we let $y = \cos\theta\tan\phi$ and $\dfrac{z}{2} = \sin\theta\tan\phi$.

   Since we want only the part that lies in front of the yz-plane, we have to ensure $x > 0$.

   So, we limit the range of ϕ to $-\pi/2 < \phi < \pi/2$.

   Therefore, the parametric equation of the given surface is:-

$$\boxed{x = \sec\phi\,, \quad y = \cos\theta\tan\phi\,, \quad z = 2\sin\theta\tan\phi\,, \quad -\pi/2 < \phi < \pi/2\,,\ \theta \in \mathbb{R}}$$

   ALTERNATIVELY,

   Let $y = u$, $z = v$. Then $4x^2 = 4 + 4u^2 + v^2 \Rightarrow x^2 = \dfrac{1}{4}(4 + 4u^2 + v^2)$.

   $\Rightarrow x = \pm\dfrac{1}{2}\sqrt{4 + 4u^2 + v^2}$. Since we want the part which lies in front of yz-plane, we have to choose $x > 0$. Therefore, the parametric equation of the given surface is:-

$$\boxed{x = \frac{1}{2}\sqrt{4 + 4u^2 + v^2}\,, \quad y = u\,, \quad z = v\,, \quad u \in \mathbb{R}\,,\ v \in \mathbb{R}}$$

3. The part of the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ that lies to the left of the xz-plane.

   Solution: To left of xz-plane, we have $y < 0$. Let $x = u$, $z = v$. Then

   $2y^2 = 1 - u^2 - 3v^2 \Rightarrow y^2 = \dfrac{1}{2}(1 - u^2 - 3v^2) \Rightarrow y = \pm\sqrt{0.5(1 - u^2 - 3v^2)}$

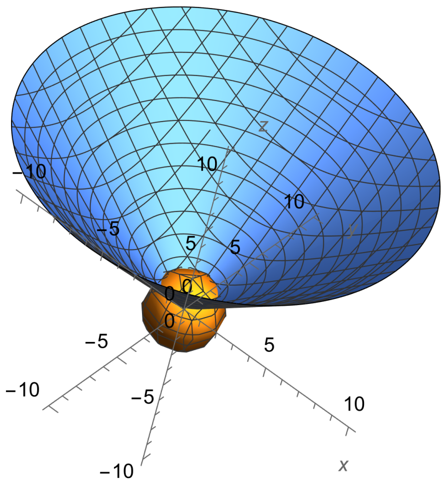
   We choose $y < 0$ since we want the part to the left of xz-plane.

Therefore, the parametric equation of the given surface is:-

$$\boxed{x = u\,,\quad y = \sqrt{0.5(1-u^2-3v^2)}\,,\quad z = v\,,\quad u, v \in \mathbb{R} \text{ such that } u^2+3v^2 \leq 1}$$

4. The part of the sphere $x^2+y^2+z^2 = 4$ that lies above the cone $z = \sqrt{x^2+y^2}$.

Solution: Shown below are both the given sphere and the given cone.



We want the part of sphere lying above the cone. So we want to have $z > \sqrt{x^2+y^2}$

This implies $z^2 > x^2+y^2$.

But since we are considering points lying on the sphere we have $z^2 = 4-(x^2+y^2)$.

Thus, we should have $4-(x^2+y^2) > x^2+y^2 \Rightarrow 2(x^2+y^2) < 4 \Rightarrow x^2+y^2 < 2$.

Now let $x = u$, $y = v$. Then $z^2 = 4-u^2-v^2 \Rightarrow z = \pm\sqrt{4-u^2-v^2}$.

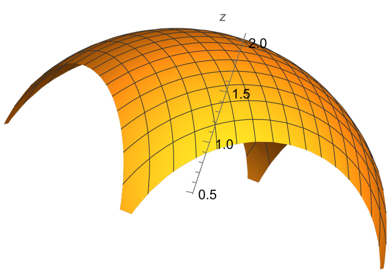
Since we want the part of sphere lying above the cone, we have to choose $z > 0$.

Thus, $z = \sqrt{4-u^2-v^2}$ and $u^2+v^2 < 2$.

Therefore, the parametric equation of the given surface is:-

$$\boxed{x = u\,,\quad y = v\,,\quad z = \sqrt{4-u^2-v^2}\,,\quad u, v \in \mathbb{R} \text{ such that } u^2+v^2 < 2}$$

The surface looks as below:-



5. The part of the cylinder $x^2 + z^2 = 9$ that lies above the xy-plane and between the planes $y = -4$ and $y = 4$.

Solution: The given cylinder has its axis to be y-axis. Let $x = u$ and $y = v$.

Since we only want the part that lies between the planes $y = -4$ and $y = 4$, we have $-4 < v < 4$.

Now, $z^2 = 9 - u^2 \Rightarrow z = \pm\sqrt{9 - u^2}$. But we choose $z > 0$ because we are considering the part of cylinder that lies above the xy-plane.

Therefore, the parametric equation of the given surface is:-

$$\boxed{x = u\,, \quad y = v\,, \quad z = \sqrt{9 - u^2}\,, \quad -3 \leq u \leq 3\,,\ -4 < v < 4}$$

The given surface looks as shown below:-