Problem 1 Find the absolute maximum and absolute minimum values of f on the given interval.

1. * $f(x) = 5 + 54\,x - 2\,x^3\,, \quad [0, 4]$.
2. *$f(x) = 3x^4 - 4x^3 - 12x^2 + 1\,, \quad [-2, 3]$.
3. $g(t) = (t^2 - 4)^3\,, \quad [-2, 3]$.
4. $f(x) = \dfrac{x}{x^2 - x + 1}\,, \quad [0, 3]$.
5. $h(p) = \dfrac{\sqrt{p}}{p^2 + 1}\,, \quad [0, 2]$.

Problem 2. A number a is called a fixed point of a function f if $f(a) = a$. Use the mean value theorem to prove that if $f'(x) \neq 1$ for all real numbers x, then f has at most one fixed point.

Problem 3. Use the mean value theorem to prove the inequality

$$|\sin a - \sin b| \leq |a - b| \quad \text{for all } a \text{ and } b\,.$$

Problem 4. Show that the equation $x^3 - 15x + c = 0$ has at most one root in the interval $[-2, 2]$.

Problem 5. Find the local maximum and minimum values of the following functions using the first derivative test.

1. * $f(x) = 1 + 3x^2 - 2x^3$.
2. * $f(x) = \dfrac{x}{x^2 + 1}$
3. $f(x) = x^4 - 2x^2 + 3$.
4. $f(x) = \sqrt{x} - \sqrt[4]{x}$.
5. $f(x) = \cos^2 x - 2\sin x\,, \quad 0 \leq x \leq 2\pi$.