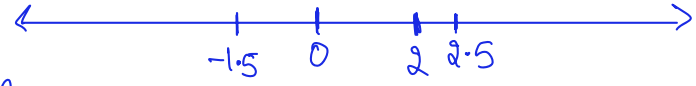
Intervals: open, closed, half-open, infinite

$(-\infty, \infty)$ → The Real number line



all the reeal numbers

open interval

$(2,3) \longrightarrow 2 < x < 3$

$[2,3) \longrightarrow 2 \le x < 3$, $(2,3] \longrightarrow 2 < x \le 3$ } half-open interval

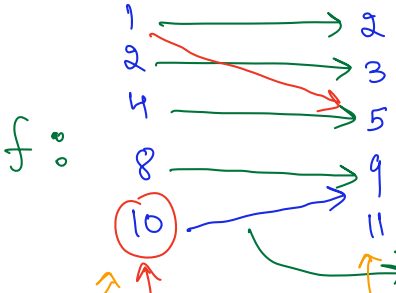
$[2,3] \longrightarrow 2 \le x \le 3$

closed interval

$(-\infty, 1) \longrightarrow x < 1$, $(-\infty, 1] \longrightarrow x \le 1$

$(2, \infty) \longrightarrow x > 2$, $[2, \infty) \longrightarrow x \ge 2$ } Infinite intervals.

Functions: independent and dependent variables, domain and range



Special property : To "every" number on the LHS we assign a "unique" number on the RHS

This is valid.

every number on LHS should be assigned something.

unique assignments implies we can write $2 = f(1)$, $3 = f(2)$, $4 = f(5)$ and so on.

use a variable x (usually) to denote LHS

use a variable y to denote RHS (usually)

$y = f(x)$

dependent variable

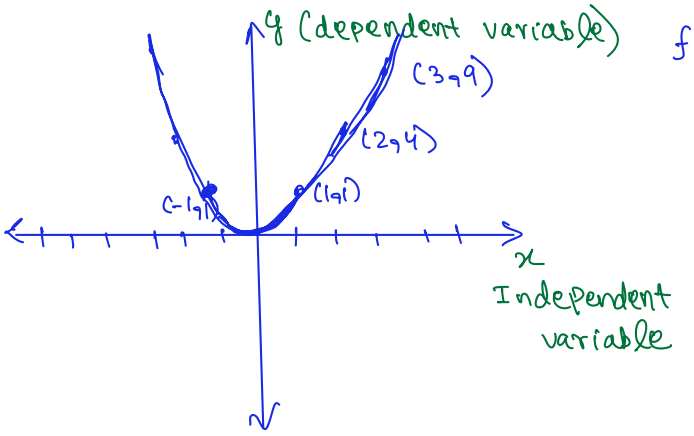
independent variable

* Collection of numbers on LHS is called Domain of f
* Collection of numbers on RHS is called range of f.

Graph of a function (Vertical line test)

denote all pairs (x, y) in a plane

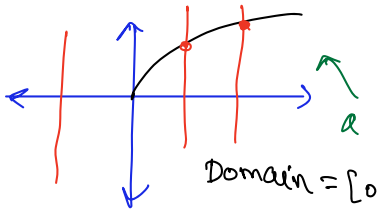
↑ a tool to visualise a function.



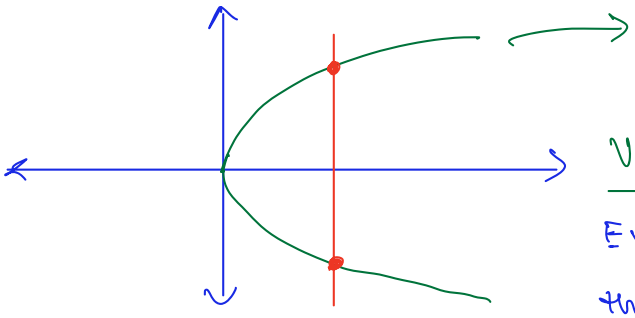
$f(x) = x^2$, $y = x^2$

Domain of $f = (-\infty, \infty)$

Range of $f = [0, \infty)$



a valid function



→ Not graph of a function.

Vertical Line test

Every vertical line should intersect the graph of a function at atmost 1 point.

Finding Domain of a given function

Example $y = \sqrt{x}$

Domain $= [0, \infty)$

Example $y = \frac{1}{x}$

Domain = Any real number except zero

$= (-\infty, 0) \cup (0, \infty)$

↑ union.

Example $y = \sqrt{x-1}$

$x - 1 \geq 0 \Rightarrow x \geq 1 \Rightarrow$ Domain $= [1, \infty)$

Example $y = \sqrt{4-x} \Rightarrow 4 - x \geq 0 \Rightarrow 4 \geq x \Rightarrow$ Domain $= (-\infty, 4]$

Alternatively multiply both sides with -1

$-1(4-x) \leq 0 \Rightarrow -4 + x \leq 0 \Rightarrow x \leq 4$

Given f, find $f(a+1)$, $f(a^2)$, $f(x+h)$ etc.

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a-b)^2 = a^2 + b^2 - 2ab$

Let $f(x) = x^2$

Find $f(a+1)$

$f(a+1) = (a+1)^2 = a^2 + 1^2 + 2(a)(1) = a^2 + 1 + 2a$

If $a=2$

$f(2+1) = f(3) = 3^2$

Composition of functions

$$(f \circ g)(x) = f(g(x))$$

let $f(x) = x^2$, $g(x) = x+1$

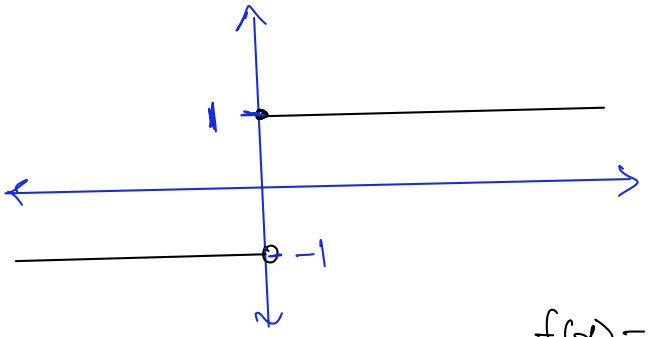
$$(f \circ g)(x) = f(g(x)) = f(x+1) = (x+1)^2$$

$g(x)$

$$= x^2 + 1 + 2x$$

Compound functions

↳ functions with different definition for different parts of the real line



$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Absolute value function

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$f(-2) = |-2| = 2 = -(-2)$

x

$f(2) = |2| = 2$