Learning objectives:

1. Applications of derivative in measuring rates of change
2. Motions of particles in physics.
3. Current in electrodynamics.
4. Marginal cost in economics.

Example 1. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$, where t is measured in seconds and s is measured in meters.

1. Find the velocity at time t.

$$v(t) = s'(t)$$
$$= 3t^2 - 12t + 9$$

2. What is the velocity after 2 s? After 4 s?

$v(2) = 3(2)^2 - 12(2) + 9 = -3$ m/s. $\longrightarrow$ Particle is moving to the left

$v(4) = 3(4)^2 - 12(4) + 9 = 9$ m/s. $\longrightarrow$ moving to the right

3. When is the particle at rest?

At rest $v(t) = 0$

$3t^2 - 12t + 9 = 0 \Rightarrow 3(t^2 - 4t + 3) = 0$

$\Rightarrow 3(t-1)(t-3) = 0 \Rightarrow t-1=0$ or $t-3=0 \Rightarrow \boxed{t = 1\text{s},\ t = 3\text{s}}$

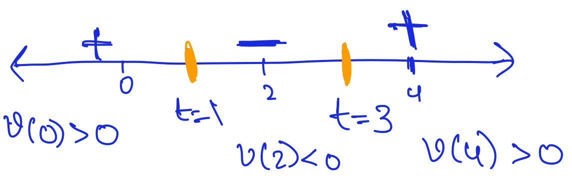
instants of rest.

4. When is the particle moving forward (that is, in the positive direction)?

Velocity is +ve

$v(t) > 0$

- Find t for when $v(t) = 0 \Rightarrow t = 1$ or $t = 3$
- Draw the number line and locate $t=1$ and $t=3$



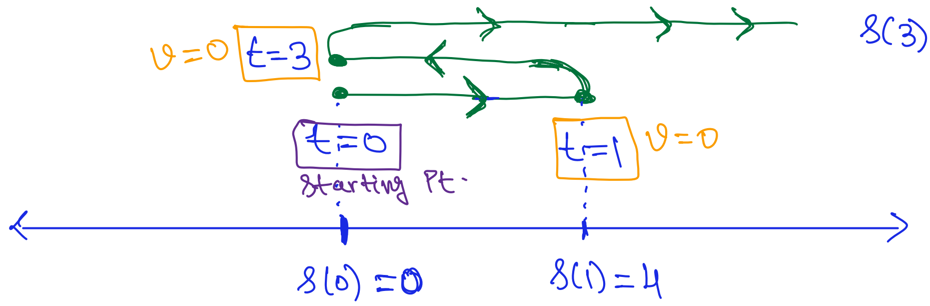
$\Rightarrow$ moving forward when $t < 1$ or when $t > 3$

5. Draw a diagram to represent the motion of the particle.

$s(t) = t^3 - 6t^2 + 9t$

$s(0) = 0, \quad s(1) = 4$

$s(3) = 27 - 54 + 27 = 0$



6. Find the total distance traveled by the particle during the first five seconds.

Cannot just say distance is $\underbrace{s(5)-s(0)}$, displacement.

distance $= |s(1)-s(0)| + |s(3)-s(1)| + |s(5)-s(3)|$

$s(5) = 5^3 - 6(5)^2 + 9(5) = 20$

$= |4-0| + |0-4| + |20-0|$

$= 4 + 4 + 20 = 28 \text{ m.}$

7. Find the acceleration at time t and after 4 s.

$$a(t) = v'(t) = \frac{d}{dt}(3t^2 - 12t + 9) = 6t - 12$$

$$a(4) = 6(4) - 12 = 12 \text{ m/s}^2$$

8. When is the particle speeding up? When is it slowing down?

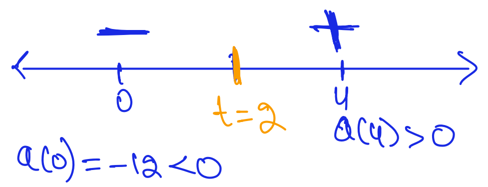
speeding up: Positive acceleration

slowing down: negative acceleration.

- Find t for which $a(t) = 0$

  $a(t) = 6t - 12 = 0 \Rightarrow 6t = 12 \Rightarrow t = 2$ s.

- Draw a number line and locate points t when $a(t) = 0$



Speeding up when $t > 2$s and slowing down when $t < 2$s

Example 2. The charge flowing through a circuit varies with times as $q(t) = 10t + 0.1\ \sin(50t + \pi)$ coulombs.

1. Find the amount of current in amperes flowing through the circuit at time t.
2. What are the maximum and minimum values of the current flowing through the circuit.

(1)

$$i(t) = q'(t) = \frac{dq}{dt}$$

$$= \frac{d}{dt}\left(10t + 0.1\ \sin(50t+\pi)\right)$$

$$= \frac{d}{dt}(10t) + 0.1\ \frac{d}{dt}\left(\sin(50t+\pi)\right)$$

$$= 10 + (0.1)\underbrace{\cos(50t+\pi)\ \frac{d}{dt}(50t+\pi)}_{\text{chain rule}}$$

$$= 10 + (0.1)50\ \cos(50t+\pi) = \boxed{10 + 5\ \underbrace{\cos(50t+\pi)}_{\text{b/w } -1 \text{ and } +1}}\ \text{Amps}$$

(2)

$$-1 \le \cos(50t+\pi) \le 1 \Rightarrow -5 \le 5\cos(50t+\pi) \le 5$$

$$10-5 \le 10+5\cos(50t+\pi) \le 10+5 \Rightarrow \underset{\text{min}}{5} \le i(t) \le \overset{\text{max.}}{15}$$

Example 3. The cost of producing x units of an item is given by $10,000 + 5x + 0.01\ x^2$ dollars. Find the cost of producing one more item after 500 items have been produced.

$$\frac{C(501) - C(500)}{501 - 500} \approx C'(500)$$

$$\underbrace{C'(x)}_{\text{marginal cost}} = \frac{d}{dx}\left(10000 + 5x + 0.01x^2\right)$$

$$= 0 + 5 + (0.01)2x = 5 + 0.02x$$

$$C'(500) = 5 + (0.02)(500) = \boxed{15 \text{ dollars.}}$$