M16600 Lecture Notes

Section 10.1: Curves Defined by Parametric Equations

■ **Section 10.1** textbook exercises, *page 685*: #5, 7, 8.

Equations such as

$$y(x) = 3e^x + x^3 \qquad \text{or} \qquad x(y) = y^2 - 1$$

describe some curves in the xy-plane.

In this section, we have ANOTHER way to describe curves in the xy-plane, called ***parametric equations***:

$$x = x(t) \qquad \text{and} \qquad y = y(t)$$

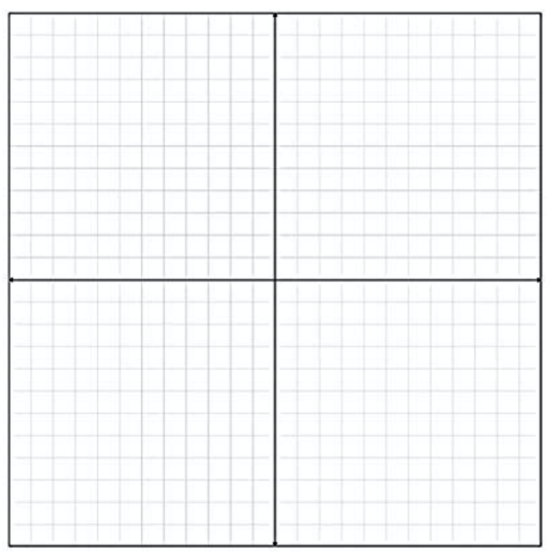
Here, t is the parameter.

Example 1: (a) Sketch the given ***parametric curves*** (i.e. curves given by *parametric equations*). Indicate with an arrow the direction in which the curve is traced as t increases.
(b) Eliminate the parameter to find a ***Cartesian equation*** (equation with only x and y) of the curve

(1) $x = t^2$ and $y = t + 1$

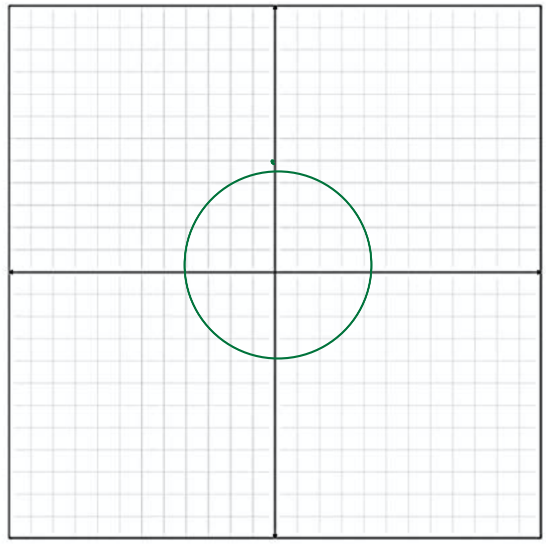
HW.

t	x	y
-3		
-2		
-1		
0		
1		
2		
3		



(2) $x = \cos t$ and $y = \sin t$, where $0 \le t \le 2\pi$.

t	x	y
0	1	0
$\pi/4$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\pi/2$	0	1
π	-1	0
$3\pi/2$	0	-1
2π	1	0



Example 2: Let $\mathcal{C}$ be the parametric curve given by $x = t^2$ and $y = t^3 - 3t$.

(a) Find the point on the curve $\mathcal{C}$ when $t = 3$.

$x = 3^2 = 9$

$y = 3^3 - 3(3) = 27 - 9 = 18$

$\Rightarrow (9, 18)$

(b) Find t at the point $(1, 2)$.

$t^2 = 1 \Rightarrow t = \pm 1$

$t^3 - 3t = 2$

Put and check

$t = \pm 1$

$\boxed{t=1}$ $\quad 1^3 - 3(1) = -2 \neq 2$

$\boxed{t=-1}$ $\quad (-1)^3 - 3(-1) = -1 + 3 = 2$

$\Rightarrow t = -1$

(satisfies $t^2 = 1$ AND $t^3 - 3t = 2$)