The method of ***Integration by Parts*** corresponds to the Product Rule in differentiation.

There is one formula you need to remember

$$\boxed{\int u\,dv = uv - \int v\,du}$$

follows from $(uv)' = uv' + u'v$

We will learn how this formula works in examples

Example 1: Find $\displaystyle\int x \sin x\,dx$

Note: u-substitution will not work for this problem.

$$I = \int \underbrace{x}_{u}\ \underbrace{\sin x\,dx}_{dv}$$

$u = x \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$

$dv = \sin x\,dx$

$v = \int \sin x\,dx = -\cos x + \underbrace{C}_{C=0}$

(Choose an antiderivative)

$$= x(-\cos x) - \int (-\cos x)\,dx$$

(RHS)

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

$$I = \int \underbrace{\sin x}_{u}\ \underbrace{x\,dx}_{dv}$$

$u = \sin x \Rightarrow du = \cos x\,dx$

$dv = x\,dx \Rightarrow v = \int x\,dx = \frac{x^2}{2}$

$$= (\sin x)\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\cos x\,dx$$

$$= \frac{1}{2}x^2\sin x - \frac{1}{2}\underbrace{\int x^2\cos x\,dx}_{\text{even more difficult}}$$

Example 2: Evaluate $\int 3x^3 \ln x \, dx$

L I A T E

Choice (priority) for u

Log.

Inverse Trigo.

Algebraic

Trigo.

Exp.

or

I L A T E

Example 3: Find $\int t^2 e^t \, dt$

Example 4: Calculate $\int_0^1 \tan^{-1} x \, dx$

Example 5: Find $\displaystyle\int e^x \sin x \, dx$